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# Assessing Hoek-Brown Parameters For Cemented Materials: A Cost-Effective And Time-Efficient Methodology

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ABSTRACT: The assessment of the strength behavior of rocks poses a challenge of intricate determination. The Hoek-Brown failure criterion for rock masses stands as the prevailing solution to this quandary and has found application in numerous global projects. However, estimating Hoek–Brown parameters is intricately tied to elaborate and time-consuming tests (e.g., triaxial tests), requiring substantial investments and complex analyses. This paper introduces a novel method for calculating Hoek-Brown parameters and establishing the nonlinear failure envelope for various types of artificially cemented materials, ensuring sample integrity. The proposed approach is then compared with real laboratory data. By utilizing basic tests like unconfined compression tests and Brazilian tests, the method estimates the maximum shear strength of materials in triaxial tests with effective confining pressures up to 400 kPa. The results from the proposed methodology exhibit a precise and conservative fit with the experimental strength results, showcasing the applicability of the approach across a broad spectrum of artificially cemented materials. Importantly, this method does not eliminate the necessity of conducting triaxial tests; however, it enables a reliable evaluation of strength parameters without the need for such tests or other intricate and time-intensive procedures.

KEYWORDS: Hoek-Brown failure criterion, shear strength, ground improvement, laboratory tests

RESUMO: A avaliação do comportamento de resistência das rochas representa um desafio de determinação intricada. O critério de ruptura de Hoek-Brown para massas rochosas surge como a solução prevalente para este dilema e encontrou aplicação em inúmeros projetos globais. No entanto, a estimativa dos parâmetros de Hoek-Brown está intimamente ligada a testes elaborados e demorados (por exemplo, testes triaxiais), exigindo investimentos substanciais e análises complexas. Este trabalho introduz um novo método para calcular os parâmetros de Hoek-Brown e estabelecer a envoltória de ruptura não linear para vários tipos de materiais artificialmente cimentados, somente quando a integridade da amostra é garantida. O método proposto é então comparado com dados reais de laboratório. Ao utilizar testes básicos como ensaios de compressão não confinada e ensaios de compressão diametral, o método estima a resistência máxima ao cisalhamento dos materiais em testes triaxiais com tensões efetivas de até 400 kPa. Os resultados da metodologia proposta exibem um ajuste preciso e conservador com os resultados de resistência experimental, demonstrando a aplicabilidade da abordagem em uma ampla gama de materiais artificialmente cimentados. Importante ressaltar que este método não elimina a necessidade de realizar testes triaxiais; no entanto, permite uma avaliação confiável dos parâmetros de resistência sem a necessidade de tais testes ou outros procedimentos intricados e demorados.

PALAVRAS-CHAVE: Critério de Ruptura de Hoek-Brown, Resistência de Materiais cimentados, Melhoramento de solos, Ensaios de Laboratório

## 1 INTRODUCTION

The determination of rock strength is a complex task, often hindered by the limitations of laboratory tests in representing larger rock masses. The Hoek-Brown failure criterion, widely accepted in the rock mechanics community, addresses this challenge by utilizing rock mass classification to estimate parameters. However, continual reassessment and refinement are necessary to accommodate practical complexities. Saroglou and Tsiambaos (2008) introduced modifications to the criterion to address anisotropy in intact rocks, while Arshadnejad and Nick (2016) proposed empirical formulations for evaluating the $m_i$ Hock-Brown parameter. Aladejare and Wang (2019) applied Bayesian methods to estimate $m_i$, highlighting the need for alternatives due to the cost and complexity of triaxial tests.

Consoli (2014) proposed a methodology for determining Mohr-Coulomb failure envelope parameters based on simple tests for artificially cemented soils, offering a practical approach without the need for complex testing. However, there is a need to explore similar methodologies for estimating the $m_i$ parameter using simplified tests from soil mechanics, as cemented soils resemble intact rock.

This study addresses this gap by conducting compressive strength tests to calculate Hoek-Brown parameters and derive failure envelopes for various types of artificially cemented soils with different treatments. Comparison with measured Mohr-Coulomb parameters and determination of shear strength parameters using Consoli's methodology reveal the potential of basic tests like unconfined compression tests for estimating maximum shear strength. The results indicate a conservative fit between the proposed methodology for estimating Hoek-Brown parameters and experimental shear strength results.

## 2 BACKGROUND

### 2.1 Hoek & Brown criterion

The Hoek-Brown failure criterion, introduced by Hoek and Brown (1980), stands as a pivotal empirical model in rock mechanics, offering insights into the intricate behavior of rock masses under stress. This criterion, derived through the analysis of extensive triaxial test data, encompasses a relationship between major and minor effective principal stresses (σ'1 and σ'3 respectively) and the uniaxial compressive strength of intact rock material (σc). It's encapsulated in Equation (1), where constants $m_i$ and *s* represent critical parameters that vary with rock properties.

$$\sigma'_1 = \sigma'_3 + \sigma_c\sqrt{m_i\frac{\sigma'_3}{\sigma_c} + s} \quad (1)$$

Essentially, the criterion serves a dual purpose: first, to accurately describe the response of intact rock material under a spectrum of stress conditions commonly encountered in practical engineering scenarios; second, to accommodate the inherent anisotropic strength behavior associated with discontinuities within rock masses. The parameter s, ranging from 0 to 1, signifies the degree of fracturing within the rock mass, with 0 indicating highly fractured material and 1 representing intact rock. Meanwhile, the constant m, always positive, encompasses a broad spectrum from 0.001 to 25, reflecting the quality of the rock mass. Notably, m is indicative of the rock's strength characteristics, with higher values associated with harder, intact rocks and lower values indicative of weaker, less coherent materials.

In determining the crucial parameter $m_i$, which is synonymous with frictional strength in the Mohr-Coulomb, Hoek and Brown (1980) advocated for its estimation through rigorous statistical analysis of triaxial tests on carefully prepared core samples. Equation (2) provides a mathematical framework for calculating $m_i$, incorporating σc and the principal stresses at failure (σ'1 and σ'3). However, given the resource-intensive nature of triaxial testing, alternative methods for estimating $m_i$ have been explored, particularly for projects where such tests are not feasible.

$$y = m \cdot x \cdot \sigma_c + s \cdot \sigma_c \quad (2)$$

Where $x = \sigma'_3$, $y = (\sigma'_1 - \sigma'_3)^2$ and $s = 1$ (for intact rocks)
For $n$ specimens the uniaxial compressive strength $\sigma_c$ and the constant $m_i$ are calculated from:

$$\sigma_c^2 = \frac{\Sigma y}{n} - \left[\frac{\Sigma xy - \left(\Sigma x \Sigma y / n\right)}{\Sigma x^2 - \left((\Sigma x)^2 / n\right)}\right] \frac{\Sigma x}{n} \quad (3)$$

$$m_i = \frac{1}{\sigma_c} - \left[\frac{\Sigma xy - \left(\Sigma x \Sigma y / n\right)}{\Sigma x^2 - \left((\Sigma x)^2 / n\right)}\right] \quad (4)$$

In such cases, rock type becomes a key determinant in estimating $m_i$, considering factors such as foliation, mineral composition, texture, and grain size. Hoek and Brown (1980) and subsequently Hoek (2007) collated data from diverse rock types and authors to establish correlations between $m_i$ and rock properties. Notably, $m_i$ is inversely related to grain size, with coarse-grained rocks exhibiting higher $m_i$ values, translating to greater friction angles, and finer-grained rocks presenting lower $m_i$ values and consequently lower friction angles.

In essence, the Hoek-Brown criterion, while empirically derived, serves as a robust tool for understanding the complex interplay of stresses within rock masses. Its ability to incorporate anisotropic behavior and provide insights into rock strength across various geological contexts underscores its significance in geotechnical engineering and rock mechanics.

## 2.2 Methodology

### *2.2.1 Procedures for estimating Mohr-Coulomb failure criterion parameters*

Consoli (2014) pioneered a methodology to assess the Mohr-Coulomb failure envelope of artificially cemented soils by analyzing splitting tensile strength (σt) and unconfined compressive strength (σc). This approach facilitated the estimation of effective cohesion intercept (c') and effective friction angle (ϕ') for any artificially cemented material, eliminating the need for triaxial testing.

The Mohr-Coulomb Failure Theory, depicted in the shear strength (τ) versus effective normal stress (σ′) space, is characterized by plotting Mohr semi-circles representing stress states at failure and drawing tangents to these semi-circles, forming the Mohr-Coulomb failure envelope. Equation (5) describes the linear relationship between shear strength (τ) and effective normal stress (σ′), incorporating effective cohesion intercept (c′) and effective friction angle (ϕ′).

Consoli (2014) provided Equations (6) and (7) based on the straight envelope defined by Equation (5) and Mohr semi-circles obtained from simple compression and splitting tensile strength tests. For stress semi-circles, minimum and maximum effective principal stresses (σ'3 and σ'1) are determined based on test conditions.

$$\tau = c' + \sigma' \cdot \tan\phi' \quad (5)$$

$$\sin\phi' = \frac{\sigma_c/2}{\left(\frac{\sigma_c}{2} + \frac{c'}{\tan\phi'}\right)} \quad (6)$$

$$\sin\phi' = \frac{-2\sigma_t}{\left(-\sigma_t + \frac{c'}{\tan\phi'}\right)} \tag{7}$$

Further investigations (Consoli et al., 2010; Consoli et al., 2013; Consoli et al., 2012a; Consoli et al., 2012b; Floss 2012, Consoli et al., 2014) confirmed the -σt/σc ratio as a scalar for various soil-cement mixtures, independent of dosage. This led to the proposal that -σt = ξ·σc, where ξ is a scalar typically ranging from -0.09 to -0.18. Incorporating this into Equations (6) and (7) yields expressions for ϕ′ and c′ (Equations 8 and 9), where ϕ′ depends only on ξ and c′ depends on ξ and σc.

$$\phi' = \arcsin\left(\frac{1+4\xi}{1+2\xi}\right) \tag{8}$$

$$c' = \frac{\sigma_c\left[1-\left(\frac{1+4\xi}{1+2\xi}\right)\right]}{2\cos\left[\arcsin\left(\frac{1+4\xi}{1+2\xi}\right)\right]} \tag{9}$$

In essence, for a given soil-cement blend, ξ remains a scalar, making ϕ′ constant and independent of σc, -σt, and dosage. However, c′ varies with ξ and σc, reflecting the nuanced interplay between soil properties and cementing agents.

*2.2.2 Proposal for estimating Hoek-Brown parameters*

As mentioned, the Hoek-Brown parameters consist of three stresses (σ'1, σ'3, and σc) and two constants ($m_i$ and s). Considering artificially cemented soils as intact rocks (s=1), the original Hoek-Brown equation (Equation 1) can be reformulated.

$$\sigma'_1 = \sigma'_3 + \sigma_c\sqrt{m_i\frac{\sigma'_3}{\sigma_c}+1} \tag{10}$$

In the unconfined compression test $\sigma'_3 = 0$, thus Eq. (10) is rewritten as:

$$\sigma'_1 = \sigma_c \tag{11}$$

Similarly, for the splitting tensile test $\sigma'_1 = -3\cdot\sigma_t$ and $\sigma'_3 = -\sigma_t$ (Jaeger et al. 2007) the Eq. (10) is rewritten as follows:

$$-\sigma_t = 0.125\,\sigma_c\left(m_i + \sqrt{m_i^2 + 16}\right) \tag{12}$$

Remembering that for artificially cemented soils the tensile/compression ratio ($\xi$) is a scalar, Eqs. (11) and (12) can be related through the Eq. (13).

$$\xi = \frac{-\sigma_t}{\sigma_c} = \frac{0.125\,\sigma_c\left(m_i+\sqrt{m_i^2+16}\right)}{\sigma_c} \tag{13}$$

By reformulating Eq. (13) it is possible to obtain the parameter $m_i$, as follows

$$m_i = \frac{(4\xi^2-1)}{\xi} \tag{14}$$

Using these formulations and having values of unconfined compression strength (σc) and diametrical tensile strength (-σt) for calculating ξ, it is possible to estimate the Hoek-Brown strength parameters and establish the corresponding rupture envelope.

Previous investigations by Consoli et al. (2010, 2013, and 2014) indicated a scalar ξ ranging from -0.09 to -0.18. Based on Equation 14, $m_i$ falls within a range from 10.75 (for ξ = -0.09) to 4.84 (for ξ = -0.18). This methodology provides a practical approach for estimating Hoek-Brown parameters in artificially cemented soils, contributing to the understanding and characterization of their strength behavior.

### 2.3 Validation of the proposed methodology

To validate and compare the methodology for determining Mohr-Coulomb and Hoek-Brown parameters, experimental data were meticulously selected from relevant literature. Studies focusing on artificially cemented mixtures, regardless of soil type, binder, or other additives, were chosen. The selection criterion ensured the integrity of the materials (s=1) and the availability of unconfined compression, splitting tensile, and triaxial compression test data.

A comprehensive dataset of 144 well-rounded triaxial tests on artificially cemented specimens was collated, considering effective confining pressures up to 400 kPa. This range aligns with realistic assumptions in engineering applications like pavement structures and foundation reinforcement.

The compiled data encompassed diverse soils with various binders and curing periods, providing a broad spectrum for evaluating the proposed model. Mixtures included sands with cement [Monterey sand (Bachus et al. 1981), Toyoura sand (Hirai et al. 1989), Osório sand (Consoli et al. 2009)], residual soils with cement (Schnaid et al. 2001), cement and fiber (Specht, 2000), lime with industrial by-products (Consoli et al. 2014), dispersive soils treated with cement (Samaniego, 2015) and fine clay (kaolin) mixed with cement (Velászquez, 2016) These mixtures exhibited variability in molding parameters such as dry specific weight, void ratio, cement content, lime content, fiber content, water content, and curing time.

To ensure representativity, all tests-maintained saturation levels above 85%, as studies (Fredlund et al. 2011 and Consoli et al. 2007) have shown that at these levels, suction's influence on strength is negligible. Analysis was divided into three phases: data collection, determination of strength parameters, and evaluation of model fitting. Mohr-Coulomb parameters were determined by constructing Mohr semi-circles on a shear strength versus effective normal stress plot, while Hoek-Brown parameters were estimated using equations derived from the proposed methodology.

The validation process demonstrates the effectiveness of the proposed methodology in estimating Hoek-Brown parameters for artificially cemented soils. By comparing theoretical and experimental values, the model's accuracy and suitability for practical engineering applications can be evaluated.

### 2.4 Verification of the proposed method

The strength parameters of Mohr-Coulomb (MC) (Consoli 2014) (c' and $\phi'$) and Hoek-Brown (HB) (this study) ($m_i$) failure envelopes were calculated using unconfined compressive strength ($\sigma c$) and tensile fracture strength ($-\sigma t$) values. The aim was to demonstrate the suitability of these methodologies for estimating the respective strength parameters of the studied mixtures without the need for triaxial or other time-consuming tests.

Figures 1a to Figure 1d, in the $\sigma'$ vs $\tau$ plane, presents the results of four different mixtures from various researchers. For comparison, the MC and HB (1980) failure envelopes were calculated using triaxial, unconfined compression, and Brazilian tests and added to the figures.

The reliability of the analyzed methodologies was evaluated through a statistical analysis of the quotient between observed ($q^{test}$) and calculated ($q^{model}$) peak strength values. Quotient values equal to or greater than one indicates conservative values. Figure 2 and Table 1 summarize the statistical analysis, showing the observed frequency and a comparison with a normal distribution curve. The analysis revealed a reasonable adjustment between the peak values of the tests and the failure envelopes. Evaluation of the mean ($\mu$), standard deviation (s) and calculation of the coefficient of variation (CoV) showed that, on average, the failure envelopes defined by the HB (this study) present a more conservative values with less variation compared to MC methodology (Consoli 2014).

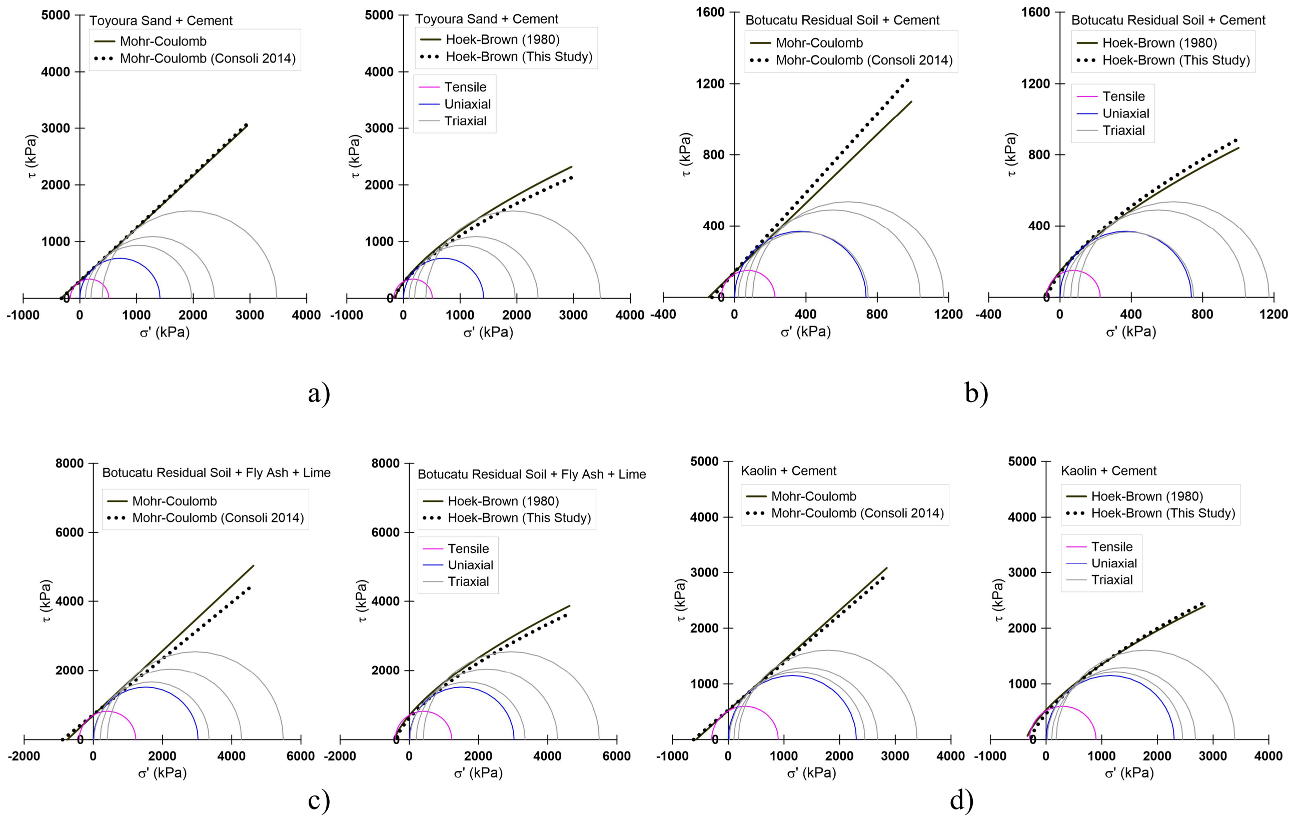


Figure 1 – Mohr-Coulomb and Hoek-Brown failure envelopes of a) Toyoura sand + cement (data from Hirai et al. 1989); b) Botucatu residual soil + cement (Data from Schnaid et al. 2001); c) Botucatu residual soil + fly ash + lime (data from Consoli et al. 2015); d) Kaolin + cement (data from Velásquez 2016)

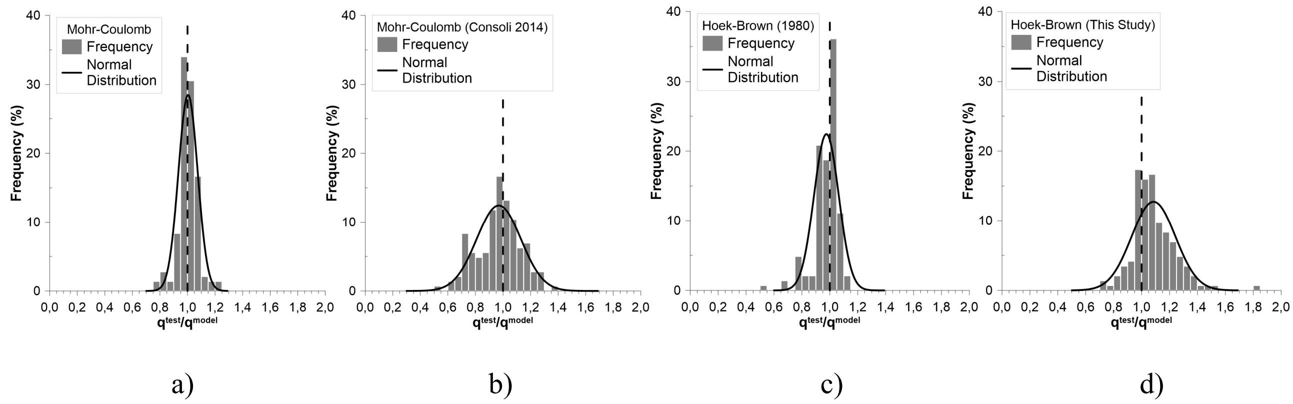


Figure 2 – Observed frequency of qtest/qmodel and normal distribution curve of failure envelopes: (a) Mohr-Coulomb, (b) Mohr-Coulomb (Consoli 2014), (c) Hoek-Brown (1980) and (d) Hoek-Brown (this study).

Table 1 – Summaries of statistical analysis results

| Methodology | Mean (μ) | Standard Deviation (s) | Coef. Variation (CoV) | Confidence Interval (95%) [min,max] |
|---|---|---|---|---|
| **Mohr-Coulomb** | 1.00 | 0.07 | 7.0% | [0.99,1.02] |
| **Mohr-Coulomb (Consoli 2014)** | 0.97 | 0.16 | 16.5% | [0.94,1.00] |
| **Hoek & Brown (1980)** | 0.98 | 0.09 | 9.1% | [0.96,0.99] |
| **Hoek & Brown (this study)** | 1.09 | 0.16 | 14.4% | [1.06,1.11] |

Analysis of the statistical data demonstrated that regardless of the failure envelope used, equivalent results were expected, with calculated values near those observed in the tests. The strength parameters estimated using HB (this study) showed slightly more conservative responses, with the average value slightly higher than one. This indicates that the methodology yields conservative safety results, aligning with standard geotechnical design practices.

Overall, the verification process confirmed the effectiveness and reliability of the proposed methodology in estimating Hoek-Brown parameters for artificially cemented soils, providing valuable insights for geotechnical engineering applications.

## 3 CONCLUSIONS

This study introduced a novel formulation for estimating the m-parameter of the Hoek-Brown rupture envelope for artificially cemented materials. The following conclusions can be drawn from the results:

- A methodology for establishing the m-parameter of the Hoek-Brown failure envelope based on unconfined compressive strength (σc) and splitting tensile strength (σt) of various treated soils was proposed and successfully validated.
- The proposed methodology is potentially applicable to a wide range of artificially cemented materials, provided that sample integrity is ensured (s=1).
- The presented methodology enables conservative estimation of Hoek-Brown's failure envelope without the need for triaxial or other complex and time-consuming tests. However, it should be noted that this study is limited to effective confining pressures (σ'3) up to 400 kPa.
- While it may be sufficient to perform only one splitting tensile strength (σt) and one unconfined compressive strength (σc) test for the same mixture, it is advisable to conduct at least three tests of each type and utilize the average of the results for applying the methodology.

Overall, this study provides a valuable contribution to the field of geotechnical engineering by offering a practical and efficient methodology for estimating Hoek-Brown parameters in artificially cemented materials, thereby facilitating the design and analysis of engineering structures.

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