XXII Congresso Brasileiro de Mecânica dos Solos e Engenharia Geotécnica
XI Simpósio Brasileiro de Engenheiros Geotécnicos Jovens
24 a 28 de agosto de 2026 – Brasília/DF

DOI: 10.47094/COBRAMSEG2026.4/81

FORM-Based Probabilistic Analysis of Jointed Rock Slopes

Higor Biondo de Assis
Ph.D. student, Department of Civil and Environmental Engineering, Faculty of Engineering Bauru, São Paulo State University, Bauru-SP, Brazil, higor.assis@unesp.br

Felipe Paiva Magalhães Vitali
Ph.D. student, Department of Geotechnical Engineering, Engineering School of São Carlos, University of São Paulo, São Carlos-SP, Brazil, felipe.vitali@usp.br

Tarcisio Barreto Celestino
Professor, Department of Geotechnical Engineering, Engineering School of São Carlos, University of São Paulo, São Carlos-SP, Brazil, tbcelest@usp.br

Caio Gorla Nogueira
Professor, Department of Civil and Environmental Engineering, Faculty of Engineering Bauru, São Paulo State University, Bauru-SP, Brazil, caio.nogueira@unesp.br

ABSTRACT: Probabilistic slope stability assessment offers a more realistic treatment of uncertainties in geotechnical parameters and modelling assumptions. This study presents a practical procedure for probabilistic analysis with relatively low computational cost by combining the Response Surface Method (RSM) with the First-Order Reliability Method (FORM). The approach is applied to a slope composed of highly weathered weak rock mass located on the Northern Ring Road in São Paulo, Brazil. Numerical simulations were performed using the Shear Strength Reduction Method within the Finite Element Method, employing average parameter values, while cohesion and friction angle of the dominant discontinuity were systematically varied according to a Central Composite Rotatable Design. Response surfaces derived from this design were integrated with FORM to estimate the probability of failure and identify the most probable combination of shear strength parameters associated with slope instability. Results from the probabilistic analysis were compared with deterministic back-analyses based on conventional factors of safety, highlighting the limitations of relying solely on deterministic approaches and illustrating the practical utility of FORM in capturing parameter uncertainties. The comparison demonstrates that FORM provides accurate and computationally efficient probabilistic predictions, serving as a robust tool for engineering decision-making in geotechnical practice.

KEYWORDS: Slope Stability; Probabilistic Analysis; FORM; Response Surface; Discontinuities.

RESUMO: A avaliação probabilística da estabilidade de taludes oferece um tratamento mais realista das incertezas associadas aos parâmetros geotécnicos e às suposições de modelagem. Este estudo apresenta um procedimento prático para análise probabilística com custo computacional relativamente baixo, combinando o Método da Superfície de Resposta (RSM) com o Método de Confiabilidade de Primeira Ordem (FORM). A abordagem é aplicada a um talude composto por maciço rochoso fraco e altamente intemperizado localizado no Rodoanel Norte, em São Paulo, Brasil. Simulações numéricas foram realizadas utilizando o Método de Redução da Resistência ao Cisalhamento no âmbito do Método dos Elementos Finitos, empregando valores médios de parâmetros, enquanto a coesão e o ângulo de atrito da descontinuidade dominante foram sistematicamente variados conforme um Desenho Rotacional Central Composto. Superfícies de resposta derivadas deste desenho foram integradas ao FORM para estimar a probabilidade de falha e identificar a combinação mais provável de parâmetros de resistência ao cisalhamento associada à instabilidade do talude. Os resultados probabilísticos foram comparados com análises determinísticas baseadas em fatores de segurança convencionais, evidenciando as limitações das abordagens determinísticas e a eficácia do FORM na captura das incertezas paramétricas.

PALAVRAS-CHAVE: Estabilidade de Taludes; Análise Probabilística; FORM; Superfície de Resposta; Descontinuidades.

1 INTRODUCTION

Rock slope stability remains a challenging problem in geotechnical engineering, particularly for highly weathered and anisotropic rock masses. The presence of foliation planes, joints, and other discontinuities introduces directional variability in strength, strongly influencing failure mechanisms and overall slope behaviour. Traditional limit equilibrium methods (LEM), widely used in practice, rely on predefined failure surfaces and simplified assumptions, which may be inadequate for complex, structurally controlled failures.

Numerical methods based on the finite element method (FEM), combined with the Strength Reduction Method (SRM), allow failure mechanisms to develop naturally through stress redistribution and progressive strength degradation. Constitutive models such as Jointed Rock and Iso-Jointed Rock enable explicit representation of oriented discontinuities, capturing anisotropic behaviour and interactions between intact rock and discontinuities.

Despite these advances, most slope stability analyses remain deterministic, neglecting uncertainties in material properties and model parameters. Reliability-based methods, particularly the First-Order Reliability Method (FORM), provide a robust framework to quantify these uncertainties. Coupled with design of experiments and response surface methodologies, FORM can efficiently estimate probabilities of failure and reliability indices, offering a comprehensive assessment of slope performance and the influence of key geotechnical parameters.

2 BRIEF DESCRIPTION OF THE CASE STUDY

This study examines the South Slope of a slope excavation along the Northern Ring Road in São Paulo, Brazil, with cut heights ranging from 20 to 30 meters, approaching a near-vertical configuration in the final stages (1H:6V). The excavation area lies within a shear zone, featuring phyllites, schists, and amphibolites, with varying degrees of weathering. The slope geology includes interbedded metabasic rocks and phyllites, with unfavourable foliation and fracture orientations, promoting planar failure mechanisms.

Localized planar failures occurred during intense rainfall in November and December 2024, predominantly within Class IV and V materials, controlled by foliation and fracture planes (Figure 1.a). Instability was absent in cuts inclined at 45° but observed in the final near-vertical cut, where the foliation dip was approximately 70° and the fracture planes dipped at around 55° (Figure 1.b), both towards the slope face. Stabilization measures were implemented but are not addressed here. The study focuses on failure mechanisms and stability analysis through simplified analytical methods, numerical modelling, and a statistical framework to estimate failure probabilities and account for geological variability.



a) b)

Figure 1. South Slope: a) foliation and b) fracture planes.

3 METHODS

The presented methodology offers a structured framework for slope stability assessment in highly weathered, anisotropic rock masses, combining classical geotechnical analysis, numerical simulations, regression modelling, and reliability-based probabilistic analysis. The workflow is organised into sequential steps, addressing parameter variability, mechanical anisotropy, and probabilistic performance. The following subsections outline the conceptual basis of the numerical simulations, parameter selection, and model implementation for the case study.

2.1 Conceptual Framework for SRM-FEM

Slope stability was assessed through two-dimensional numerical simulations combining FEM with the Shear Strength Reduction (SRM) technique. This approach progressively reduces material strength parameters until loss of equilibrium occurs, providing a direct evaluation of the Strength Reduction Factor (SRF), which can be interpreted as the conventional factor of safety (FS). Failure mechanisms develop naturally from stress redistribution and progressive strength degradation, yielding a realistic representation of slope behaviour.

The rock mass was represented using constitutive models capable of capturing anisotropic behaviour associated with oriented discontinuities, specifically the Jointed Rock and Iso-Jointed Rock models (Cai and Horii, 1993). While the Jointed Rock model accounts solely for the strength of discontinuities, the Iso-Jointed Rock model additionally considers the contribution of the intact rock matrix. In both cases, isotropic elastic stiffness is assumed, with anisotropy introduced exclusively through strength parameters. A detailed discussion on the applicability of these models to slope and tunnel problems is presented by Stelzer (2015).

The SRM-FEM simulations generate a consistent dataset of SRF values for various parameter combinations. These data are then used to construct response surfaces linking key input parameters to slope stability outcomes. In probabilistic analyses, selected discontinuity parameters are treated as random variables, enabling the estimation of probability of failure and reliability indices through reliability-based methods.

2.2 Central Composite Rotatable Design (CCRD)

A Central Composite Rotatable Design (CCRD) was employed to systematically investigate the influence of key input parameters on slope stability. CCRD is a structured design of experiments widely used to explore the effects of multiple variables while maintaining a minimal number of simulation runs. It defines discrete combinations of selected factors, in this case the cohesion and friction angle of rock discontinuities, within their calibrated ranges, providing uniform coverage of the parameter space.

This approach allows the construction of response surfaces, which serve as surrogate models to approximate slope behaviour across the range of plausible parameter values. By employing statistically independent input variables, the CCRD ensures that interactions and nonlinear effects between cohesion and friction angle can be captured efficiently. Such structured experimental planning is particularly useful in geotechnical applications where numerical simulations are computationally expensive, and parameter uncertainty is significant.

The CCRD derives from a full factorial 2-level ($FF2^k$) design, supplemented by a rotated $FF2^k$ and a central point to enable rotatability and the estimation of curvature. The combination of these elements forms the CCRD, providing efficient coverage of the factor space and allowing accurate construction of response surfaces.

Based on the SRF results obtained from the CCRD simulations, multivariate linear regression was performed to develop polynomial models approximating slope stability as a function of the selected parameters. Polynomial terms were tested and adjusted following the principles of design of experiments (DOE), with model adequacy assessed using the F-Snedecor test and the statistical significance of each independent variable evaluated with the t-Student test. The F-test validates the overall polynomial regression model, while the t-test identifies which input variables significantly influence the slope response. Upon successful verification of both tests, the resulting polynomial can be employed as a surrogate model or response

surface, enabling efficient probabilistic analysis and sensitivity assessment. The regressions are performed using the Least Squares Method, with the regression tool in Excel applied for the model fitting.

2.3 First-Order Reliability Method (FORM)

FORM is a reliability method that belongs to the category of transformation methods. It involves solving an optimisation problem to find the design point ($\boldsymbol{x}^*$), which minimises the reliability index (β), subject to limit state equation $g(\boldsymbol{x}) = 0$. The limit state function, also known as the performance function, mathematically defines the event of interest for which the probability of occurrence is to be calculated, based on the marginal probabilities of the input random variables. In the context of slope stability, it is defined in the physical space as $g(\boldsymbol{X}) = FS(\boldsymbol{X}) - 1.00$ or $g(\boldsymbol{X}) = SRF(\boldsymbol{X}) - 1.00$, and $g(\boldsymbol{X}) \leq 0$ corresponds to the failure domain, while g(**X**) > 0 corresponds to the survival domain. The limit state function relates the input variables to determine the critical conditions for failure or stability of the system and is essential for the probabilistic analysis of safety. The random variable realisations ($\boldsymbol{x}$) are transformed isoprobabilistically from the physical space ($\mathbb{X}$) to the uncorrelated standard normal space ($\mathbb{Y}$) through $\boldsymbol{y} = T(\boldsymbol{x})$, allowing $\boldsymbol{y}^*$ and β to be iteratively defined within a predefined tolerance. Convergence is checked for both β and the Euclidean norm of the design point vector ($\|\boldsymbol{y}\|$). Once β converges, its equivalent failure probability (pf) is calculated using Equation 1:

$$pf = \Phi(-\beta) \tag{1}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution. The procedure follows the HLRF algorithm (Hasofer and Lind, 1974; Rackwitz and Fiessler, 1978), providing β, pf, $\boldsymbol{x}^*$, and the direction cosines vector ($\boldsymbol{\alpha}$). The vector $\boldsymbol{\alpha}$ indicates the director cosines of the random variables, with $\boldsymbol{\alpha}^2$ representing the sensitivity factors. Further details can be found in Melchers and Beck (2019), Assis & Nogueira (2023, 2024).

2.4 Integrated Methodology with RASS

The methodology integrates Central Composite Rotatable Design, regression-based response surface modelling, and FORM to provide a computationally efficient framework for slope stability analysis. The CCRD is used to define a set of input parameters, such as cohesion and friction angle of discontinuities, within their calibrated ranges, from which a response surface is constructed. Multivariate regression analysis is then applied to the response surface data, with statistical tests such as F-Snedecor and t-Student ensuring the validity of the regression models.

Reliability analysis is carried out using FORM, which incorporates uncertainties in the input parameters and calculates key reliability metrics, such as β and pf. These analyses are processed using the Assessment applied to Slope Stability (RASS) software, ensuring computational efficiency. This integrated approach enables a probabilistic evaluation of slope stability, supporting more informed and reliable decision-making in geotechnical practice, particularly in complex environments such as urbanised areas with highly weathered rock masses.

3 PROBLEM SETTING AND RESULTS

The problem is set up using a two-dimensional numerical model developed in PLAXIS 2D, with FEM combined with SRM for slope stability analysis. The material is modelled as highly weathered phyllite, using both the Jointed Rock and Iso-Jointed Rock constitutive models to represent anisotropic behaviour due to foliation planes and oriented discontinuities. These models assume isotropic elastic stiffness, with anisotropy introduced through strength parameters.

The slope geometry consists of a critical cross-section with a total excavation height of 20 m, including an 8 m high subvertical cut at its base. The model domain extends 125 m horizontally and 40 m in depth, with lateral boundaries restrained in the normal direction and the base fully fixed. Finite element mesh uses 16-

node elements with fourth-order interpolation to provide sufficient accuracy for analyses involving significant material and geometric non-linearity.

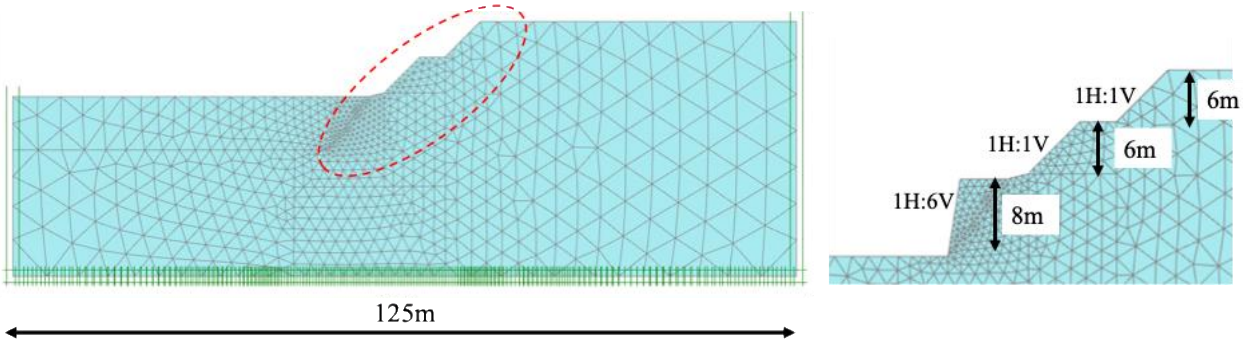
The analysis was conducted in three stages: first, the initial in situ stress state was established with gravitational loading; second, excavation of the subvertical portion of the slope was simulated; and finally, the SRF was determined once numerical convergence was reached. The SRF serves as the conventional FS in geotechnical practice, with failure identified by numerical non-convergence and significant displacement increments as strength parameters are progressively reduced. The mean parameters for the rock mass and discontinuities are summarised in Table 1.

Table 1 – Geotechnical parameters: mean values for deterministic analysis.

Modelo	E (MPa)	ν	ϕ_{intact} (∘)	c_{intact} (kPa)	ϕ_{disc} (∘)**	c_{disc} (kPa)**
Jointed Rock	150	0,25	-	-	30º	27
Iso-Jointed Rock	150	0,25	35º	50*	30º	27

* For the intact rock, a tension cut-off value of 5 kPa was adopted.
** For the discontinuities, an orientation of 54° was assumed, with no tensile strength allowed (tension cut-off = 0).

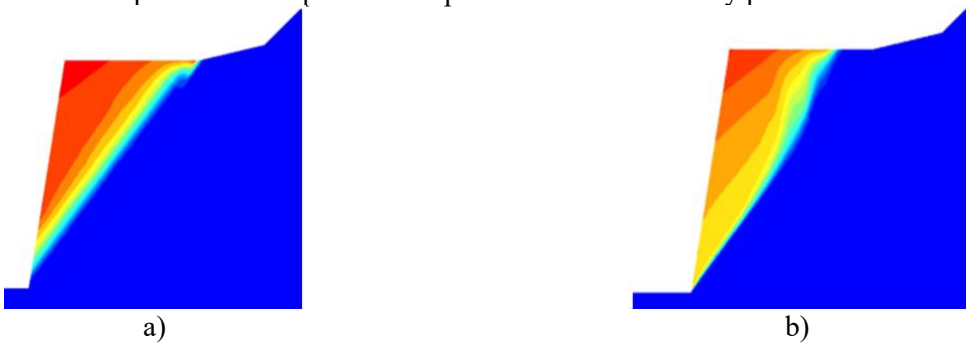
Figure 2 illustrates the results of the strength reduction analysis for the base cases, highlighting the contours of incremental plastic displacement magnitude, which define the failure surface, for both constitutive models. For the Jointed Rock model, the failure surface is essentially planar (Figure 3.a), with an inclination of 54°, resulting in a strength reduction factor of 1.28, in good agreement with the results obtained using the Culmann method.



a) b)

Figure 2. SRM model: a) finite element mesh; b) geometry after excavation.

In contrast, for the Iso-Jointed Rock model, the failure surface initially develops as an approximately linear feature, influenced by the adopted tensile strength, and subsequently evolves into a circular shape until it intersects the discontinuity plane dipping at 54° (Figure 3.b). In this case, the strength reduction factor decreases to 1.15, reflecting the influence of the intact rock parameters. This behaviour is considered more representative of field conditions, as the rock mass is highly fractured, and the failure mechanism is governed by both the rock mass response and the presence of preferential discontinuity planes.



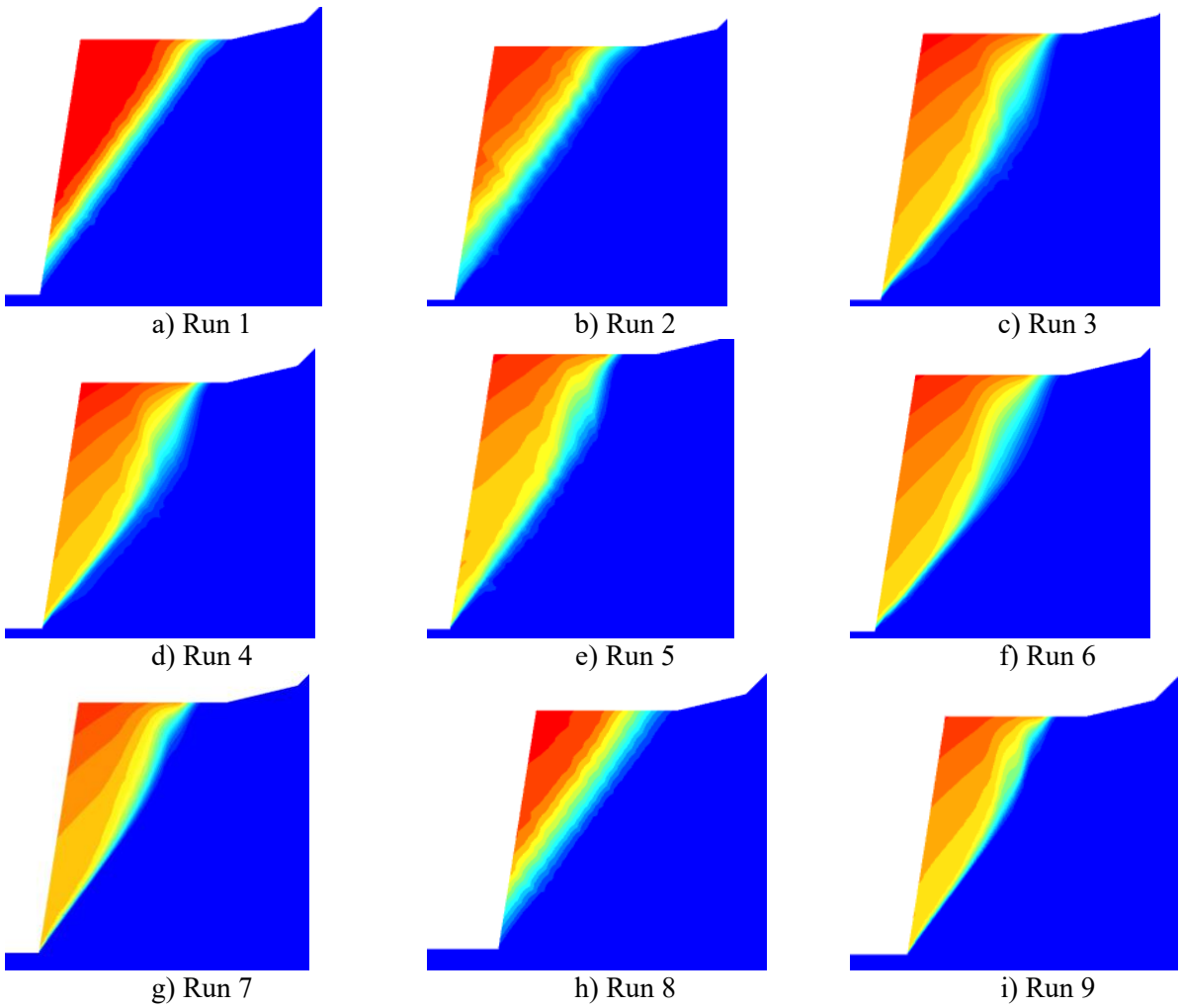
a) b)

Figure 3. Typical differences between the slip surfaces: a) Jointed Rock and b) Iso-Jointed Rock model.

For cases in which the factor of safety is lower than 1, its evaluation within the finite element framework was based on the unloading level at which numerical non-convergence occurs during the excavation stage. In this approach, when the finite element mesh is deactivated to simulate excavation, the previously balanced nodal forces are preserved and gradually reduced. This unloading process is continued until numerical non-convergence is reached, which is associated with the development of a failure mechanism. The corresponding unloading level at this condition is then interpreted as the factor of safety of the system.

Table 2 – CCRD runs for SRF evaluation.

	Physical space		Levels	
Runs	c_{disc} (kPa)	$\tan \phi_{disc}$	c_{disc}	$\tan \phi_{disc}$
1	9.766	0.454	-1	-1
2	9.766	0.700	-1	+1
3	44.234	0.454	+1	-1
4	44.234	0.700	+1	+1
5	27.000	0.750	0	+1.41
6	51.300	0.577	+1.41	0
7	27.000	0.404	0	-1.41
8	2.700	0.577	-1.41	0
9	27.000	0.577	0	0



a) Run 1 b) Run 2 c) Run 3
d) Run 4 e) Run 5 f) Run 6
g) Run 7 h) Run 8 i) Run 9

Figure 4. Failure surfaces for SRF evaluation from CCRD runs using the Iso-Jointed Rock model.

The CCRD was applied to systematically explore the parameter space around the calibrated mean values, according to Table 2. Numerical simulations were conducted for each experimental run in PLAXIS using the SRM technique within the FEM. Both Jointed Rock and Iso-Jointed Rock constitutive models were adopted to capture anisotropic behaviour, including foliation planes and oriented weaknesses. The failure surfaces for the Iso-Jointed Rock model are shown in Figure 4.

The resulting *SRF* were then fitted to first-order polynomial response surfaces (Equation 2, Table 3, Figure 5-6), which were validated through ANOVA and residual analysis to ensure they accurately represent the slope response.

$$g(\boldsymbol{X}) = \widehat{SRF}(\boldsymbol{X}) - 1.00 = [a_0 + a_1 \cdot \boldsymbol{x_1} + a_2 \cdot \boldsymbol{x_2}] - 1.00$$
$$= [a_0 + a_1 \cdot \boldsymbol{c_{disc}} + a_2 \cdot \boldsymbol{tan\phi_{disc}}] - 1.00 \quad (2)$$

Table 3 – Coefficients of linear regression model.

	Model	
Regression coefficients	Jointed Rock	Iso-Jointed Rock
a_0	0.077240679911626	0.111898704200553
a_1	0.030433827674368	0.022419259023991
a_2	0.708533363359114	0.757756926509746

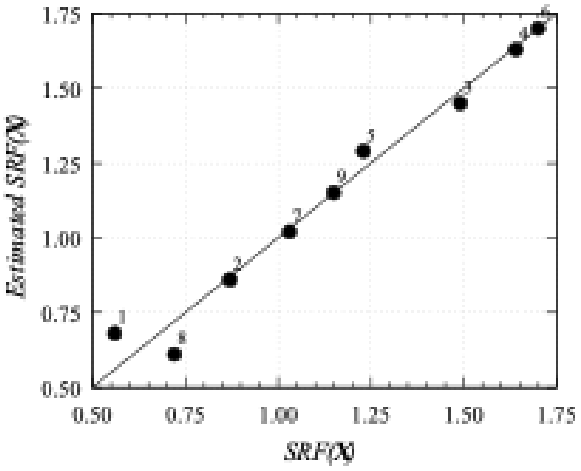
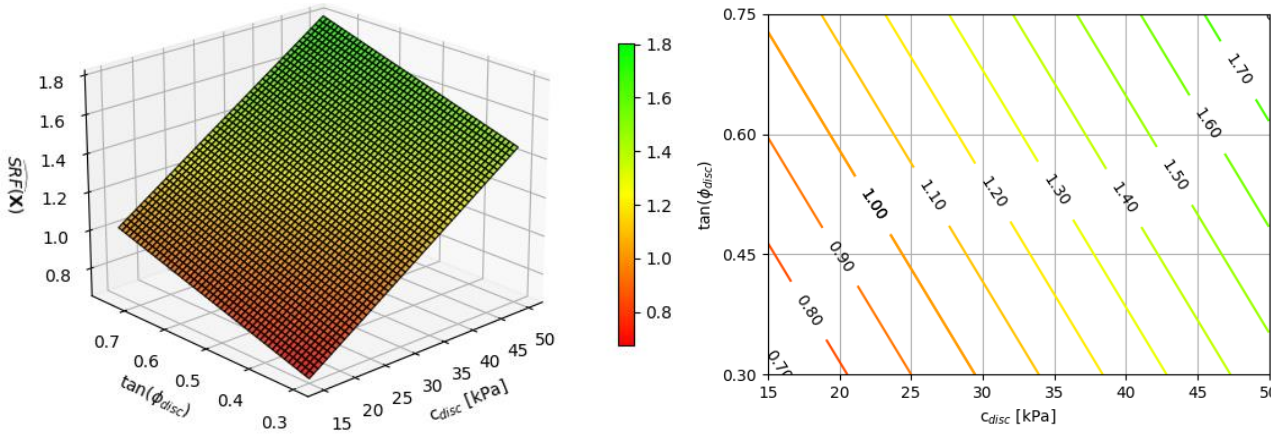


Figure 5. Iso-Jointed Rock modelled using multivariate linear regression, with a coefficient of determination $R^2 = 0.9764$.



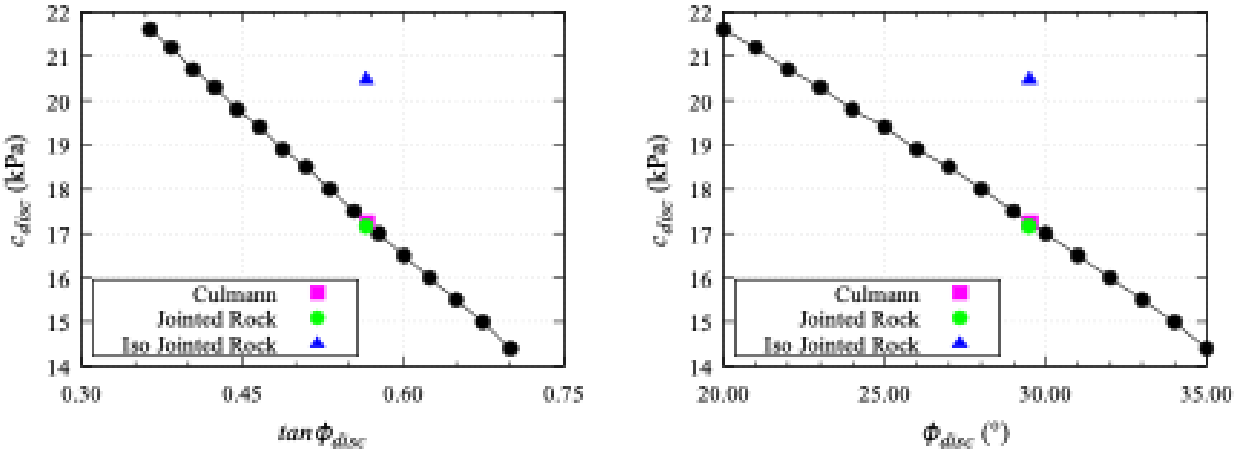
a) b)

Figure 6. Iso-Jointed Rock model: a) planar response surface and b) contour levels of SRF as a function of c_{disc} and $tan\phi_{disc}$.

The response surfaces were used as limit-state functions in the FORM, allowing estimation of the probability of failure and identification of the most probable combination of shear strength parameters corresponding to the limit state. For comparison of the probabilistic responses, complementary analyses were performed using the Culmann method directly coupled with FORM, fixing the failure surface passing through the toe of the slope and inclined at 54°. Monte Carlo simulations were also employed in all cases, for comparison with the β and pf results from FORM. The compiled results are presented in Table 3, highlighting the wealth of information provided by FORM when applied appropriately.

Table 3. Summary of results and processing time.

			Limit State Function		
			DC Culmann	RSM Jointed Rock	RSM Iso-Jointed Rock
Deterministic		$FS(\overline{\boldsymbol{x}})$	1.304	-	-
		$SRF(\overline{\boldsymbol{x}})$	-	1.280	1.150
		$\widehat{SRF}(\overline{\boldsymbol{x}})$	-	1.308	1.154
Probabilistic	MCS	n_s	5000	5000	5000
		n_f	549	534	996
		pf	10.98%	10.68%	19.92%
		β	1.23	1.24	0.84
		$COVpf$	4.03%	4.09%	5.48%
		Δt (s)	0.0336	0.0322	0.0327
	FORM	β	1.22	1.23	0.83
		pf	11.17%	10.90%	24.09%
		x_1^*	17.25	17.16	20.47
		x_2^*	0.5665	0.5654	0.5658
		$\alpha_{x_1}^2$	97.75%	97.32%	94.52%
		$\alpha_{x_2}^2$	2.25%	2.68%	5.48%
		Δt (s)	0.0007	0.0007	0.0017



a) b)

Figure 7. Comparison of deterministic back-analysis using the Culmann method (black) with the design points from probabilistic analyses via FORM (coloured).

The set of results provides valuable insights into the mechanisms under analysis. It is observed that, in all cases, the responses from FORM are highly consistent with those obtained from MCS. Moreover, the results from the Culmann method and the Jointed Rock model are quite similar due to the simplifications inherent in these models, which impose stricter constraints on the geometries of the failure surfaces. In contrast, the Iso-

Jointed Rock model coupled with FORM, produces different geometries for the failure surfaces, as expected, leading to higher failure probabilities.

The incorporation of both the Jointed Rock and Iso-Jointed Rock models, when used with FORM, significantly enhances the reliability of the slope stability analysis. These models allow for more accurate representation of geological complexities, offering a more comprehensive understanding of the slope's behaviour. The results from FORM also provide deeper insight into the sensitivity of the pf to variations in the random variables of the model, such as cohesion and friction angle.

Regarding the design point, an interesting application is seen in back-analysis. Unlike back-analyses based purely on deterministic methods, which provide a range of parameter combinations leading to slope failure, FORM allows for the identification of the combination of parameters that results in FS or SRF equal to 1, with the highest probability of occurrence. This advantage makes FORM particularly beneficial when used alongside the Jointed Rock and Iso-Jointed Rock models, as it enhances the accuracy and reliability of the predictions compared to MCS alone. Furthermore, the results from FORM indicate that, for the analysed cases, 94% or more of the failure probability is governed by the c_{disc}, while less than 6% is influenced by the $tan\phi_{disc}$. Another important observation is the comparison of processing times between FORM and MCS. Although the processing times for MCS are not prohibitive, FORM provides a more computationally efficient alternative without compromising the accuracy of the analysis.

4 CONCLUSIONS

The results underscore the robustness of the FORM method in conjunction with the Jointed Rock and Iso-Jointed Rock models for probabilistic slope stability analysis. The integration of these models significantly improves the representation of the geological complexities of the slope and enhances the reliability of the predictions. This combined approach allows for a more detailed understanding of the controlling factors for slope stability, offering better sensitivity to parameter variability and improving the overall accuracy of the analysis.

In addition to being time-efficient, FORM, when applied alongside the Jointed Rock and Iso-Jointed Rock models, provides a substantial advantage over traditional deterministic methods and MCS. The ability to determine the most probable parameter combinations leading to failure further strengthens the predictive power of the model, facilitating more informed decision-making in geotechnical engineering projects.

Thus, the combined use of FORM, and the Jointed Rock and Iso-Jointed Rock models offers a significant improvement in the reliability and efficiency of slope stability analyses, making it a powerful tool for addressing complex geotechnical challenges.

ACKNOWLEDGEMENTS

We gratefully acknowledge the financial support from *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior* (CAPES) and *Conselho Nacional de Desenvolvimento Científico e Tecnológico* (CNPq) - Brasil. We also appreciate the support and collaboration of Via Appia Concessões S.A.

REFERENCES

Assis, H. B., Nogueira, C.G., (2023) Comparative study of deterministic and probabilistic critical slip surfaces applied to slope stability using limit equilibrium methods and the First-Order Reliability Method. *Soils and Rocks*, 46(2), p.e2023013522.

Assis, H. B., Nogueira, C. G. (2024) Probabilistic Analysis of Slope Stability: System Reliability in Series with FORM as an Alternative to Monte Carlo Simulations. In: XXI Brazilian Congress of Soil Mechanics and Geotechnical Engineering (*XXI COBRAMSEG*), Balneário Camboriú-SC.

Cai M., Horii, H. (1993) A constitutive model and FEM analysis of jointed rock masses. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics*, Vol.30, Issue 4, pp. 351-359.

Hasofer, A. M., LIND, N. C. (1974) Exact and Invariant Second-Moment Code Format. *Journal of the Engineering Mechanics Division*, v. 100, n. 1, p. 111–121.

Melchers, R. E., Beck, A. T. (2018). *Structural Reliability Analysis and Prediction*. John Wiley & Sons.

Montgomery, D. C. (2017). *Design and analysis of experiments*. John Wiley & Sons.

Rackwitz, R., Fiessler, B. (1978) Structural reliability under combined random load sequences. *Computers & Structures*, v. 9, n. 5, p. 489–494.

Rodrigues, M. I., Iemma, A. F. (2014) *Experimental design and process optimization*. CRC Press.

Stelzer, M. (2015) *Numerical Studies on the PLAXIS Jointed Rock Model*, Master Thesis, TU Graz.

Vitali, F. P. M., Vitali, O. P. M., Bobet, A., & Celestino, T. B. (2024) FEM modeling requirements for accurate analysis of highly nonlinear shallow tunnels. *Soils and Rocks*, 47(1), e2024000923.