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Surrogate Equation for Rapid Stability Assessment of Layered Cohesive Slopes

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ABSTRACT: This study proposes a surrogate equation for rapid estimation of the factor of safety (FS) of two-layered cohesive slopes under undrained conditions. The formulation eliminates the iterative slip-surface searches typically required in limit equilibrium analyses. The model was derived from a parametric database of 384 simulations performed using the Morgenstern–Price method implemented in SLOPE/W. The stability problem is expressed using two dimensionless parameters: the stability number $N_{s,1} = c_{u1}/(\gamma_1 H_1)$and the strength ratio $R_{cu} = c_{u1}/c_{u2}$. Parametric analyses revealed two governing failure mechanisms: shallow failures confined to the upper layer and deep failures involving both layers. Their transition is defined by a critical strength ratio dependent on slope inclination. Based on normalized stability relationships, a closed-form equation is proposed to directly estimate the factor of safety. The surrogate model reproduces the numerical solutions with excellent accuracy ($R^2 \approx 1.0$), with approximately 92% of predictions within ±5% of the reference results. The proposed formulation provides a computationally efficient tool for preliminary design, parametric analysis, and rapid stability screening of layered cohesive slopes.

KEYWORDS: Slope Stability, Layered Soils, Surrogate Model, Limit Equilibrium Method, Factor of Safety.

1 INTRODUCTION

Slope stability analysis is a fundamental problem in geotechnical engineering because slope failures can lead to severe economic losses, environmental damage, infrastructure disruption, and risks to human safety. The primary objective of slope stability assessment is the determination of the factor of safety (FS), defined as the ratio between the available shear resistance and the mobilized shear stress along a potential failure surface (Abramson et al., 2002; Duncan & Wright, 2005).

Several analytical and numerical approaches have been developed to evaluate slope stability. Among these, the limit equilibrium method (LEM) remains the most widely adopted technique in engineering practice due to its conceptual simplicity and satisfactory accuracy for many practical problems (Fredlund & Krahn, 1977; Duncan & Wright, 2005). In LEM analyses, the soil mass is divided into slices and equilibrium conditions are imposed to estimate the factor of safety. Classical formulations include the methods of Bishop (1955), Janbu (1973), and Morgenstern and Price (1965), the latter satisfying both force and moment equilibrium and therefore being regarded as one of the most rigorous LEM formulations.

Despite their widespread use, LEM analyses require iterative procedures to identify the critical slip surface. Modern stability programs typically evaluate hundreds or thousands of potential slip surfaces in a single analysis. Although such computations are feasible for individual slopes, they may become inefficient when large parametric studies, probabilistic analyses, or regional-scale assessments are required (Cheng et al., 2007). Consequently, simplified analytical tools—such as stability charts and regression-based equations—remain valuable for preliminary design and rapid engineering assessments.

Early stability charts were primarily developed for homogeneous slopes using dimensionless parameters such as the stability number (Taylor, 1937; Chen, 1975). However, natural and engineered slopes commonly

exhibit layered soil profiles, in which contrasts in mechanical properties significantly influence the failure mechanism and overall stability (Duncan & Wright, 2005; Griffiths & Lane, 1999). In layered systems, failure may occur as shallow mechanisms confined to the upper soil layer or as deeper mechanisms involving multiple layers (Zheng et al., 2005).

Recent studies have improved the understanding of failure mechanisms in layered slopes. Guo and Griffiths (2020) demonstrated that the transition between shallow and deep failure mechanisms in two-layer undrained slopes is governed by a critical strength ratio that depends on slope geometry. Qian et al. (2014) developed stability charts for two-layered purely cohesive soils using finite-element limit analysis, providing reference solutions for comparison with limit equilibrium approaches. Additional numerical investigations have also emphasized the influence of strength contrasts and stratigraphy on the geometry of failure surfaces and the resulting stability conditions (Chatterjee & Krishna, 2019).

Despite these advances, simplified analytical tools capable of directly estimating the factor of safety for layered slopes remain limited. Most available approaches rely on numerical simulations that require iterative calculations and specialized software. Regression-based surrogate models provide an attractive alternative by approximating numerical solutions through explicit equations derived from extensive parametric analyses (Cheng et al., 2007). Recently, Sampa and Schorr (2024) proposed a closed-form stability equation for homogeneous dry slopes, demonstrating the potential of regression-based formulations for rapid stability assessment.

In this context, the objective of the present study is to develop a surrogate model for estimating the factor of safety of two-layered cohesive slopes under undrained conditions. The formulation is derived from an extensive parametric database generated using the Morgenstern–Price limit equilibrium method implemented in SLOPE/W. The stability problem is expressed using dimensionless parameters that capture slope geometry and soil strength contrast. Based on normalized stability relationships, a closed-form equation is proposed to directly estimate the factor of safety without iterative calculations or slip-surface searches. Unlike classical stability charts developed for homogeneous slopes, the proposed formulation provides a direct analytical solution for layered cohesive slopes while explicitly accounting for the strength contrast between soil layers.

2 PROBLEM DEFINITION AND NUMERICAL DATABASE

The slope configuration analyzed in this study consists of two horizontal cohesive soil layers (Figure 1). The upper layer has thickness H_1, while the lower layer thickness H_2 was selected sufficiently large to avoid boundary effects and ensure that potential slip surfaces remain fully contained within the computational domain. A thickness ratio $H_2/H_1 = 2.857$ was adopted to prevent the lower boundary from influencing the critical failure mechanism.

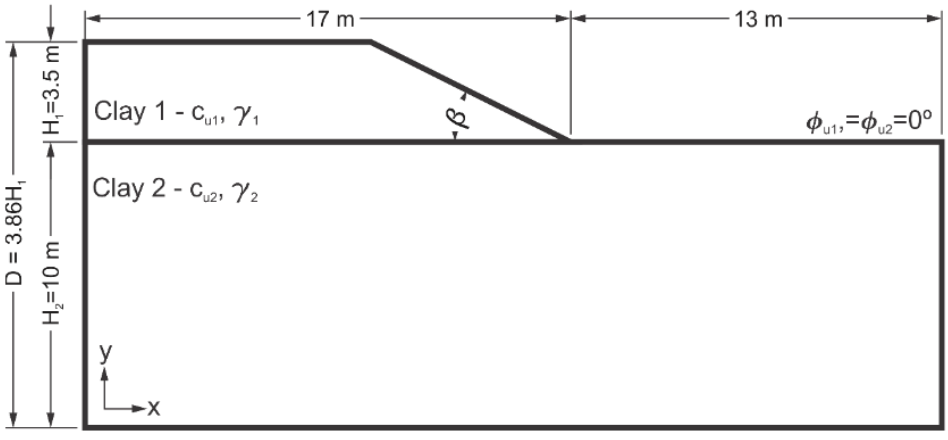


Figure 1. Slope geometry and definition of parameters used in two-layered cohesive slopes.

Both soil layers are assumed homogeneous, isotropic, and purely cohesive, representing short-term undrained conditions. Under these conditions, soil strength is characterized by the undrained shear strength $c_u$with friction angle $\phi = 0°$. Slope inclinations considered in the analyses range from 10°to 85°, covering a wide range of practical configurations.

Preliminary parametric analyses confirmed that variations in γ_2 do not influence FS within the investigated range, consistent with classical undrained stability formulations where unit weight effects are captured through $N_{s,1}$ normalization.

The factor of safety was calculated using the Morgenstern–Price limit equilibrium method implemented in SLOPE/W, which satisfies both force and moment equilibrium conditions. Circular slip surfaces were assumed and the soil mass was discretized into 30 slices. The critical slip surface was identified using the Entry–Exit search algorithm.

A total of 384 numerical simulations were performed to construct the parametric database used for the development of the surrogate model.

3 DIMENSIONLESS PARAMETERS

To generalize the stability behavior of layered slopes, the results were expressed using dimensionless parameters. This approach allows the stability problem to be analyzed independently of specific values of soil strength or slope height.

The first parameter is the stability number $N_{s,1}$, which represents the ratio between the undrained shear strength of the upper layer and the gravitational stresses acting on the slope.

$$N_{s,1} = \frac{c_{u1}}{\gamma_1 H_1} \tag{1}$$

The second parameter is the strength ratio R_{cu}, which quantifies the mechanical contrast between the two soil layers:

$$R_{cu} = \frac{c_{u1}}{c_{u2}} \tag{2}$$

where c_{u1} and c_{u2} denote the undrained shear strengths of the upper and lower layers, respectively.

Using $N_{s,1}$ and R_{cu}, the stability problem can be described in normalized form, allowing the results of the parametric analyses to be generalized for different combinations of slope height, soil strength, and unit weight.

4 FAILURE MECHANISMS

The analysis of the simulation results revealed two distinct failure mechanisms (Figure 2) governing the stability of two-layered cohesive slopes: 1) shallow failure mechanism (SFM), in which the critical slip surface remains entirely confined within the upper soil layer, and the stability is controlled primarily by the strength of the upper soil; and 2) deep failure mechanism (DFM), in which the slip surface penetrates the lower layer, mobilizing the shear strength of both soils and producing a deeper rotational failure.

The transition between these mechanisms is governed by a critical strength ratio, defined as

$$R_{cu,t} = \begin{cases} (1.34 \tan\beta + 0.19131)/(\tan\beta + 0.930071) \text{ for } \tan\beta < 2.145 \\ 1 \text{ for } \tan\beta \geq 2.145 \end{cases} \tag{3}$$

For $R_{cu} < R_{cu,t}$, shallow failure mechanisms dominate, whereas for $R_{cu} > R_{cu,t}$, deep failures involving both soil layers are more likely to occur.

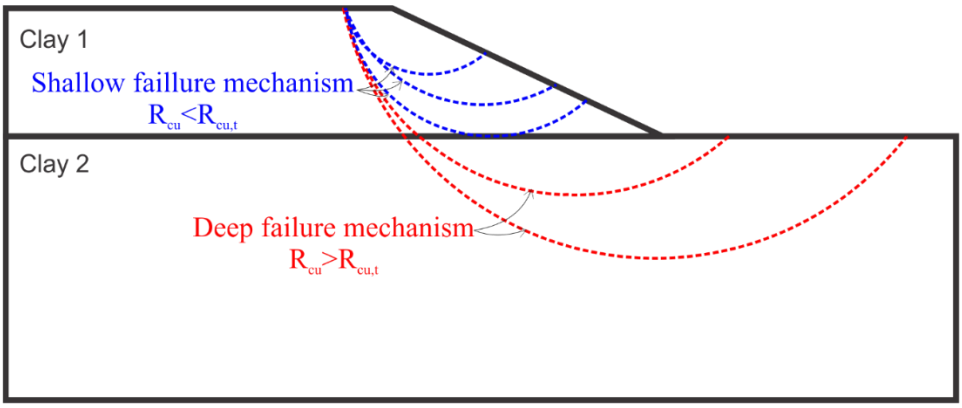


Figure 2. Illustrative examples of shallow and deep failure mechanisms in two-layered cohesive slopes.

5 SURROGATE MODEL FORMULATION

Analysis of the normalized stability results shown in Figure 3 revealed a consistent relationship between the factor of safety and the governing dimensionless parameters. When the simulation results were expressed in terms of the stability number $N_{s,1}$, the strength ratio R_{cu}, and the slope inclination β, the data exhibited a systematic trend that could be represented by a compact analytical expression.

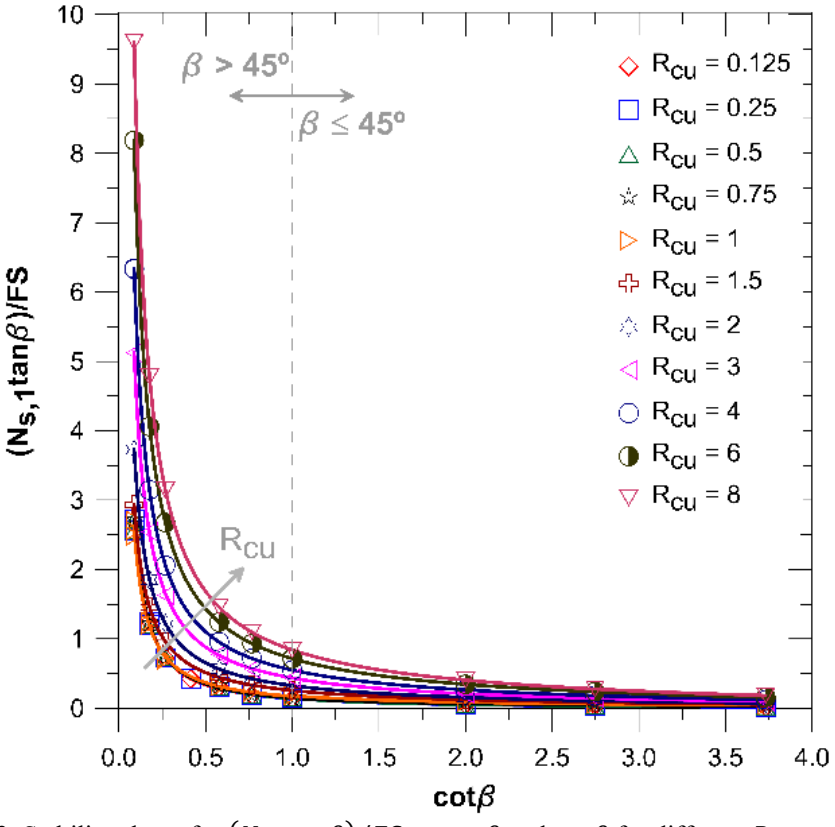


Figure 3. Stability charts for $(N_{s,1}\tan\beta)/FS$ versus β and $\cot\beta$ for different R_{cu}.

Based on this observation, a surrogate formulation was developed to approximate the numerical stability solutions obtained from the parametric database. The normalized relationship between the governing parameters can be written as

$$\frac{N_{s,1}\tan\beta}{FS} = \alpha + \lambda(\cot\beta)^{\eta} \tag{4}$$

which leads to the explicit expression for the factor of safety

$$FS = \frac{N_{s,1}\tan\beta}{\alpha + \lambda(\cot\beta)^{\eta}} \tag{5}$$

where the coefficients α, λ, and $\eta$are functions of the strength ratio R_{cu}.

Equation (5) constitutes the core of the proposed surrogate model and provides a direct estimate of the factor of safety without the need for iterative slip-surface searches. The coefficients were obtained through nonlinear regression analysis. The regression procedure was performed separately for different ranges of slope inclination in order to accurately capture the variation of stability behavior for gentle and steep slopes.

5.1 Stability Coefficient α

The coefficient α represents the base component of the normalized stability relationship and primarily captures the influence of the strength contrast between the soil layers. Regression analysis indicated that its variation with the strength ratio R_{cu} is nonlinear and depends on the slope inclination range.

For slopes with $\beta \leq 45°$

$$\alpha_{\beta\leq 45°} = \begin{cases} -0.000376231272 - 0.04110515449\,R_{\mathrm{cu}} + 0.0898229976{R_{\mathrm{cu}}}^{2} \rightarrow R_{\mathrm{cu}} \leq 0.5 \\ -0.03141278 + 0.033812632\Big/\left(1 + \left(\dfrac{R_{\mathrm{cu}}}{0.864689}\right)^{6.589680847}\right) \rightarrow R_{\mathrm{cu}} \leq 1.5 \\ 0.04175963226 - 0.06901897751R_{\mathrm{cu}} + 0.01387189347{R_{\mathrm{cu}}}^{2} \rightarrow R_{\mathrm{cu}} \leq 3 \\ 0.27206308 - 0.16930029R_{\mathrm{cu}} + 0.025800368{R_{\mathrm{cu}}}^{2} - 0.001363547{R_{\mathrm{cu}}}^{3} \rightarrow R_{\mathrm{cu}} > 3 \end{cases} \tag{6}$$

For slopes with $\beta > 45°$

$$\alpha_{\beta>45°} = \begin{cases} -0.03487332 - 0.21935951R_{\mathrm{cu}} + 0.5986073{R_{\mathrm{cu}}}^{2} - 0.3853962{R_{\mathrm{cu}}}^{3} \rightarrow R_{\mathrm{cu}} \leq 0.75 \\ \dfrac{-0.02597697 + 0.05419579R_{\mathrm{cu}} - 0.03382571{R_{\mathrm{cu}}}^{2} + 0.0060637{R_{\mathrm{cu}}}^{3}}{-0.6787970428 + R_{\mathrm{cu}}} \rightarrow R_{cu} \leq 3 \\ -0.019722 + 0.0098429R_{\mathrm{cu}} - 0.0014336{R_{\mathrm{cu}}}^{2} + 4.919580 \cdot 10^{-5}{R_{\mathrm{cu}}}^{3} \rightarrow R_{\mathrm{cu}} > 3 \end{cases} \tag{7}$$

5.2 Stability Coefficient λ

The coefficient λ controls the influence of the geometric parameter $\cot\beta$ in the surrogate formulation and therefore reflects the effect of slope inclination on the stability response. Similar to α, its variation with the strength ratio $R_{cu}$was determined through nonlinear regression of the numerical database.

For slopes with $\beta \leq 45°$

$$\lambda_{\beta\leq 45°} = \begin{cases} 0.1393483 + 0.12460477R_{\mathrm{cu}} - 0.40837816{R_{\mathrm{cu}}}^{2} + 0.35398599{R_{\mathrm{cu}}}^{3} \rightarrow R_{\mathrm{cu}} \leq 0.75 \\ 0.00822789 + 0.19383423R_{\mathrm{cu}} + 0.0031561536{R_{\mathrm{cu}}}^{2} - 0.005297767{R_{\mathrm{cu}}}^{3} \rightarrow R_{\mathrm{cu}} \leq 3 \\ -0.4031586 + 0.42757877R_{\mathrm{cu}} - 0.052712741{R_{\mathrm{cu}}}^{2} + 0.002588045{R_{\mathrm{cu}}}^{3} \rightarrow R_{\mathrm{cu}} > 3 \end{cases} \tag{8}$$

For slopes with $\beta > 45°$

$$\lambda_{\beta>45^\circ} = \begin{cases} \dfrac{0.45307315 + 0.66110171R_{cu} - 1.11804156{R_{cu}}^2 + 0.6259963835{R_{cu}}^3}{2.649772421 + R_{cu}} \rightarrow R_{cu} \leq 1 \\ 3.171477 - 3.26640162 \Big/ \left(1 + \left(\dfrac{R_{cu}}{27.43813}\right)^{0.7299422177}\right) \rightarrow R_{cu} > 1 \end{cases} \quad (9)$$

5.3 Stability Coefficient η

The coefficient η controls the curvature of the relationship between the normalized stability term and the geometric parameter $\cot\beta$. This parameter was found to vary smoothly with the strength ratio and therefore can also be expressed through piecewise regression functions.

For slopes with $\beta \leq 45^\circ$

$$\eta_{\beta\leq45^\circ} = \begin{cases} \dfrac{-0.6009979008 + 1.484787545R_{cu} - 1.068551901{R_{cu}}^2}{0.4612101622 - 1.249244622R_{cu} + {R_{cu}}^2} \rightarrow R_{cu} \leq 1 \\ -0.7924623333 - 0.1132066072\,R_{cu} + 0.0340071732\,{R_{cu}}^2 \rightarrow R_{cu} \leq 2 \\ -0.900062 + 0.00845961R_{cu} + 0.00020047{R_{cu}}^2 - 7.460119 \cdot 10^{-5}{R_{cu}}^3 \rightarrow R_{cu} > 2 \end{cases} \quad (10)$$

For slopes with $\beta > 45^\circ$

$$\eta_{\beta>45^\circ} = \begin{cases} \dfrac{-1.916033454 + 0.9596684209R_{cu}}{1.659064186 - 0.08204390275R_{cu} - 1.71952164{R_{cu}}^2 + {R_{cu}}^3} \rightarrow R_{cu} \leq 1 \\ \dfrac{39.42720518 - 58.4526004R_{cu}}{-53.46415798 + 77.4239903R_{cu} - 7.901230184{R_{cu}}^2 + {R_{cu}}^3} \rightarrow R_{cu} \leq 4 \\ -1.014175399 + 0.003410549375R_{cu} - 0.0001508339625\,{R_{cu}}^2 \rightarrow R_{cu} > 4 \end{cases} \quad (11)$$

5.4 Computational Procedure

The proposed surrogate model can be implemented using the following procedure: 1) define the slope geometry H_1 and inclination β; 2) obtain soil parameters c_{u1}, c_{u2}, and unit weight γ_1; 3) compute the dimensionless parameters $N_{s,1}$ and R_{cu}; 4) evaluate the coefficients α, λ, and η using Eqs. (6)-(11); 5) substitute these coefficients into Eq. (5) to calculate the factor of safety.

This procedure enables rapid stability estimation without iterative calculations or slip-surface searches, making the model suitable for preliminary design and large-scale parametric analyses.

6 MODEL VALIDATION

The predictive performance of the surrogate model was evaluated by comparing the factors of safety estimated using Eq. (5) with the numerical results obtained from the limit equilibrium analyses performed in SLOPE/W using the Morgenstern–Price method.

Figure 4 presents the comparison between the factors of safety predicted by the surrogate formulation and those obtained from the numerical simulations included in the parametric database. The results show excellent agreement between the two approaches, with a coefficient of determination $R^2 \approx 1.0$, indicating that the proposed equation accurately reproduces the stability solutions generated by the numerical analyses.

Approximately 92% of prediction fall within ±5% of the numerical values across the investigated range of slope inclinations and strength ratios. This level of agreement demonstrates that the surrogate formulation effectively captures the influence of the governing dimensionless parameters and provides reliable estimates of the factor of safety for two-layered cohesive slopes.

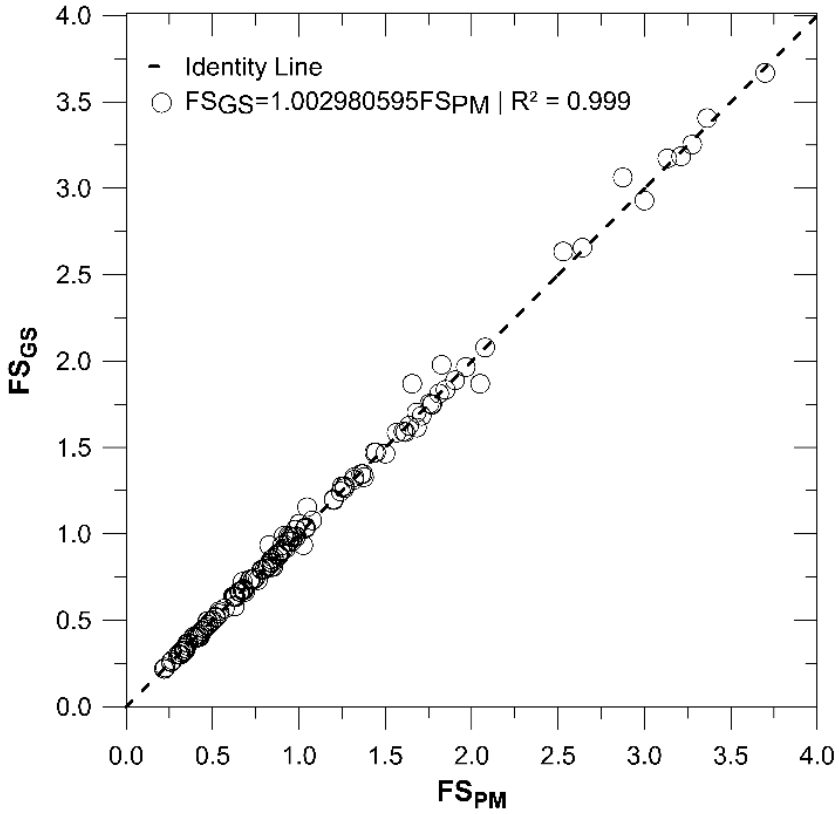


Figure 4. Comparison between the FS_{PM} and FS_{GS}.

8 CONCLUSIONS

This study developed a surrogate formulation for estimating the factor of safety of two-layered cohesive slopes under undrained conditions. The approach replaces the iterative slip-surface search typically required in limit equilibrium analyses with a direct analytical expression derived from parametric numerical simulations.

The stability behavior of layered slopes was successfully represented using two dimensionless parameters: the stability number and the strength ratio between the soil layers. The analyses revealed the existence of two distinct failure mechanisms: shallow failures confined to the upper layer and deep failures involving both soil layers. A critical strength ratio dependent on slope inclination was identified to define the transition between these mechanisms.

The proposed surrogate equation provides a direct estimate of the factor of safety and reproduces the numerical stability solutions with high accuracy across the investigated parameter range. Because the formulation avoids iterative calculations, it enables rapid evaluation of slope stability and is particularly suitable for preliminary design and large parametric studies.

The model is applicable to two-layered cohesive slopes under short-term undrained conditions with circular failure surfaces, inclinations $10° \leq \beta \leq 85°$, strength ratios $0.125 \leq R_{cu} \leq 8$, and lower-layer thickness $H_2/H_1 \geq 2.86$. Limitations include undrained conditions, purely cohesive soils, circular failure surfaces, and the absence of seepage, external loads, or soil heterogeneity. For critical applications or conditions outside these bounds, verification through advanced numerical methods is recommended.

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