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Surrogate Equation for Rapid Stability Assessment of Layered Frictional Slopes

Naloan Coutinho Sampa
Professor, Federal University of Santa Catarina, Florianópolis, Brazil, naloan.sampa@ufsc.br

Humberto Cosmos Ferreira de Sales Silveiro da Silva Volpato
Undergrad. Student, Federal University of Santa Catarina, Florianópolis, Brazil, humberto.cosmos@grad.ufsc.br

Katharyna do Carmo Macedo
Undergrad. Student, Federal University of Santa Catarina, Florianópolis, Brazil, katharynadc@gmail.com

ABSTRACT: Efficient evaluation of slope stability is essential for preliminary design, regional hazard assessment, and large parametric studies. Conventional limit equilibrium methods provide reliable estimates of the factor of safety (FS), but require iterative searches for the critical slip surface, which can become computationally demanding when numerous slope configurations must be analyzed. This study presents a surrogate formulation for estimating the factor of safety of dry two-layered frictional slopes. The model was developed from a parametric database of 192 stability analyses performed using the Morgenstern–Price limit equilibrium method implemented in SLOPE/W. Three dimensionless parameters describing slope geometry, strength contrast between layers, and unit weight ratio were introduced to characterize the governing variables of the problem. Analysis of the results revealed two distinct stability regimes associated with shallow and deep failure mechanisms. A combined dimensionless parameter was then proposed to unify the stability behavior and collapse the numerical results into consistent trends. Based on these relationships, piecewise equations were derived to estimate the factor of safety directly without iterative slip-surface searches. Validation against the numerical dataset demonstrates excellent predictive accuracy, with a coefficient of determination close to unity and a standard deviation of approximately 2.8%. The proposed formulation provides a computationally efficient and physically interpretable tool for rapid stability assessment of layered frictional slopes while preserving the fundamental failure mechanisms governing their stability.

KEYWORDS: Slope Stability, Layered Soils, Surrogate Model, Limit Equilibrium Method, Factor of Safety.

1 INTRODUCTION

Slope stability analysis is a central topic in geotechnical engineering due to its direct implications for the safety and performance of infrastructure such as embankments, dams, open-pit mines, and transportation corridors. Failures of natural or engineered slopes can result in significant economic losses and human casualties, making reliable stability assessment essential for engineering practice (Sloan 2013; Goswami and Chakraborty 2020; Wubalem 2022).

The stability of a slope is commonly evaluated through the factor of safety (FS), defined as the ratio between resisting and driving shear stresses acting along a potential failure surface (Bishop 1955; Abramson et al. 2002). Several analytical and numerical approaches have been developed to estimate this parameter, including limit equilibrium methods (LEM), finite element methods, and limit analysis techniques (Duncan 1996; Griffiths and Lane 1999; Li et al. 2016). Among these approaches, LEM formulations remain the most widely used in engineering practice due to their conceptual simplicity and satisfactory accuracy (Fredlund and Krahn 1977; Wu et al. 2018). Rigorous formulations such as the Morgenstern-Price method satisfy both force and moment equilibrium but require iterative searches for the critical slip surface.

Simplified analytical solutions and stability charts have been developed to reduce this computational effort (Taylor 1937; Bishop and Morgenstern 1960; Michalowski 2002). However, most available formulations assume homogeneous soil conditions, whereas natural slopes frequently consist of layered soil

profiles with contrasting mechanical properties. In such cases, stratification can strongly influence both the stability of the slope and the governing failure mechanism (Chatterjee and Krishna 2018; Sarkar and Chakraborty 2021).

This study develops a surrogate model for estimating the factor of safety of dry two-layered frictional slopes. The formulation is derived from a comprehensive database of limit equilibrium analyses using the Morgenstern-Price method and expressed through dimensionless parameters representing slope geometry, strength contrast, and unit weight ratio. The resulting equations allow direct estimation of the factor of safety without iterative analysis while also providing insight into the governing failure mechanism.

2 PROBLEM DEFINITION AND NUMERICAL DATABASE

The problem investigated consists of a two-layer slope composed of purely frictional soils. The adopted geometry is illustrated in Figure 1. The slope comprises an upper layer with thickness H_1 =3.5 m overlying a lower layer with thickness $H_2 = 10.0$ m, resulting in a thickness ratio $H_2/H_1 = 2.857$. This configuration was selected to ensure that the lower layer behaves as a sufficiently deep foundation, preventing boundary effects from influencing the critical slip surface. The adopted ratio exceeds the threshold identified by Sampa and Schorr (2024), beyond which deeper layers exert negligible influence on the stability of homogeneous slopes.

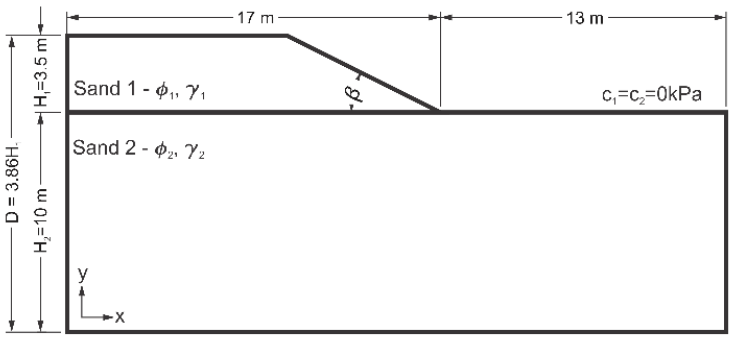


Figure 1. Geometry of the slope and definition of parameters.

Both soils were assumed homogeneous and isotropic and modeled using the Mohr-Coulomb failure criterion with zero cohesion, representing dry cohesionless materials. The friction angles of the upper and lower layers (ϕ_1 and ϕ_2) were varied independently between 10° and 40°, covering a wide range of frictional soil conditions. The unit weights of the two layers were also varied between 14 kN/m³ - 26 kN/m³ to evaluate the influence of density contrast between strata.

Slope inclination β was varied from 10° to 85°, encompassing configurations ranging from gentle embankments to steep natural slopes. The combination of geometric and strength parameters defined a broad parametric space capable of capturing the transition between shallow and deep failure mechanisms.

All stability analyses were performed using the Morgenstern-Price limit equilibrium method implemented in the software SLOPE/W. This method satisfies both force and moment equilibrium and is widely regarded as one of the most rigorous formulations within the limit equilibrium framework (Fredlund and Krahn 1977). The sliding mass was discretized into 30 vertical slices, and the critical circular slip surface was identified using the Entry-Exit search method with 100 trial surfaces.

A total of 192 slope configurations were analyzed by systematically varying slope inclination and the friction angles of both soil layers. The resulting dataset provides the basis for the development of the surrogate stability model presented in the following sections.

3 DIMENSIONLESS PARAMETERS

To generalize the stability behavior and reduce the dimensional complexity of the problem, three dimensionless parameters were introduced. These parameters describe the relative influence of layer strength contrast, slope geometry, and unit weight differences on slope stability, and capture the key physical variables

controlling slope stability in layered frictional systems and provide a consistent framework for analyzing the numerical dataset.

The strength contrast parameter (Equation 1) describes the frictional strength contrast between the upper and lower layers.

$$N_\phi = \frac{\tan \phi_1}{\tan \phi_2} \tag{1}$$

The geometry-strength parameter (Equation 2) represents the ratio between available shear strength in the upper layer and the driving shear stress associated with the slope inclination.

$$M_{\phi,\beta} = \frac{\tan \phi_1}{\tan \beta} \tag{2}$$

The unit weight ratio is shown in Equation 3, where γ_1 and γ_2 are the unit weights of the upper and lower layers.

$$R_\gamma = \frac{\gamma_1}{\gamma_2} \tag{3}$$

4 DEVELOPMENT OF THE SURROGATE MODEL

Figure 2 illustrates the variation of the factor of safety (FS) with the friction angle of the upper layer for different combinations of lower-layer strength and slope inclination. The results confirm that both the frictional strength contrast and slope geometry significantly influence stability behavior.

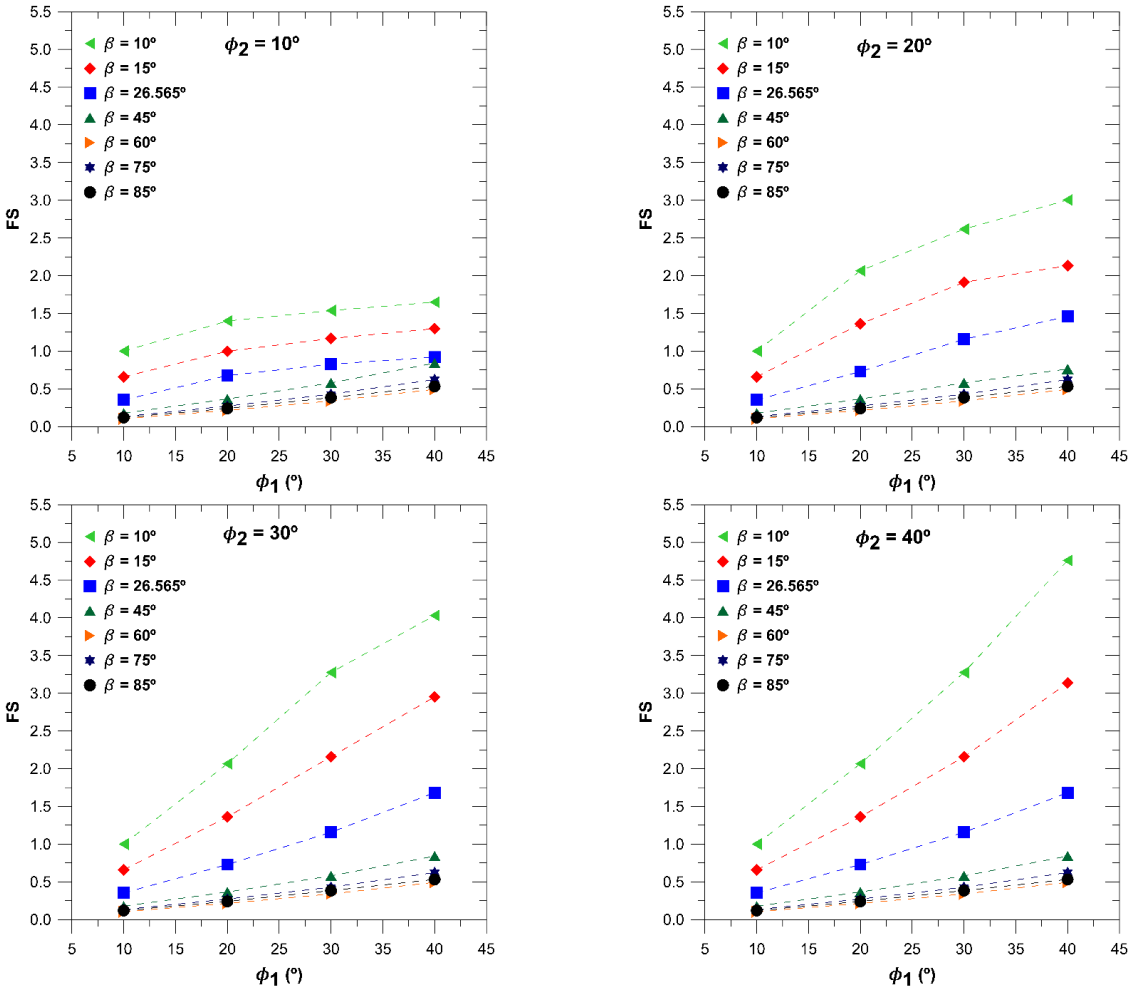


Figure 2. Variation of FS as a function of ϕ_1 for different values of ϕ_2 and β (R_γ = 1).

Analysis of the numerical dataset revealed systematic relationships between the factor of safety and the dimensionless parameters N_ϕ, $M_{\phi,\beta}$ and R_γ. As expected, the factor of safety increases with increasing friction angles and decreases as the slope inclination becomes steeper.

For slopes steeper than approximately 45°, instability was observed across most strength combinations. In this regime the failure surface tends to remain confined within the upper layer, indicating that slope geometry and upper-layer strength dominate the stability response.

For gentler slopes, however, the stability behavior becomes more complex. The critical slip surface may extend into the lower layer depending on the relative strength of the strata, indicating a stronger interaction between the two layers.

These observations reveal the existence of two distinct stability regimes: 1) steep slopes, where stability is primarily governed by slope geometry and the strength of the upper layer; 2) gentle slopes, where stability depends on the interaction between both soil layers and their relative strength.

To unify the stability behavior observed in the numerical results, a combined dimensionless parameter (K) was introduced:

$$K = N_\phi(\cos\beta)^5 \tag{4}$$

The exponent applied to $\cos\beta$ was determined empirically through regression analysis and was found to provide the best collapse of the numerical results into a single trend. Figure 3 shows the relationship between the stability coefficient μ and the strength contrast parameter for different slope inclinations.

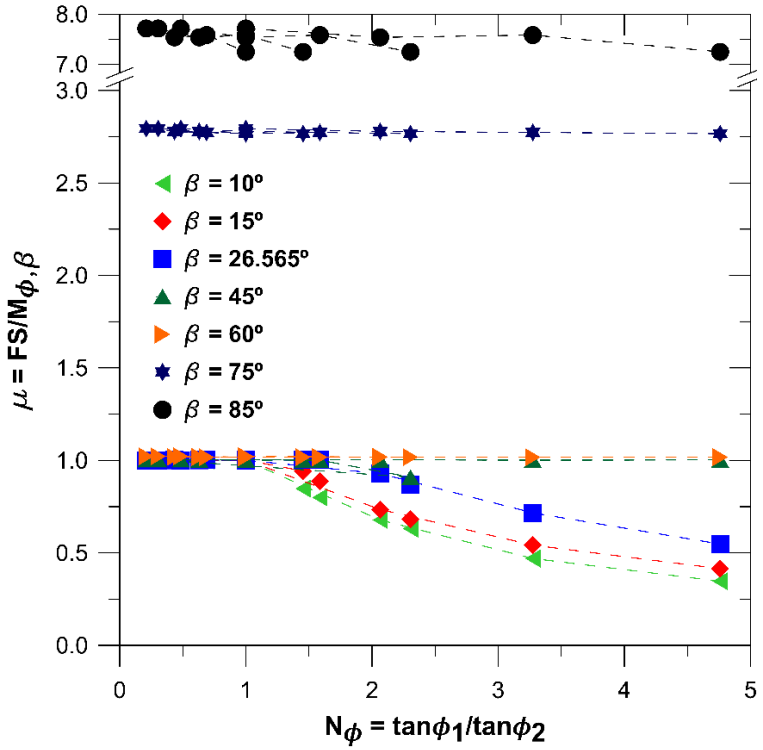


Figure 3. Variation of μ as a function of N_ϕ for different slope inclinations β (R_γ = 1).

When the factor of safety is expressed in terms of the combined parameter K, the numerical results converge into two well-defined relationships corresponding to the stability regimes identified above. This parameter therefore governs both the magnitude of the factor of safety and the transition between shallow and deep failure mechanisms.

5 SURROGATE EQUATIONS FOR THE FACTOR OF SAFETY

Based on the trends observed in the numerical dataset (Figure 3), the factor of safety can be expressed as

$$FS = \mu M_{\phi,\beta} \tag{5}$$

For slopes gentler than 40° ($\beta \leq 40°$), the stability response depends on the interaction between both soil layers. In this regime the coefficient μ is expressed as a polynomial function of the combined parameter K for $K > 1$:

$$\mu = \frac{FS}{M_{\phi,\beta}} = \begin{cases} 1.00153, & K \leq 1 \\ A + BK + CK^2 + DK^3 + EK^4, & K > 1 \end{cases} \tag{6}$$

where the coefficients A–E depend on the unit-weight ratio R_γ.

For $R_\gamma \leq 1$,

$$\begin{aligned} A &= -5.841R_\gamma^2 + 18.342R_\gamma - 11.445 \\ B &= 11.578R_\gamma^2 - 30.092R_\gamma + 18.688 \\ C &= -6.659R_\gamma^2 + 16.813R_\gamma - 10.534 \\ D &= 1.578R_\gamma^2 - 3.946R_\gamma + 2.492 \\ E &= -0.1331R_\gamma^2 + 0.3314R_\gamma - 0.2106 \end{aligned} \tag{7}$$

For $R_\gamma > 1$,

$$\begin{aligned} A &= -19.220R_\gamma^4 + 75.994R_\gamma^3 - 103.564R_\gamma^2 + 61.750R_\gamma - 13.576 \\ B &= -2.094R_\gamma^4 + 8.436R_\gamma^3 - 15.143R_\gamma^2 + 11.968R_\gamma - 3.545 \\ C &= 18.658R_\gamma^4 - 70.743R_\gamma^3 + 100.018R_\gamma^2 - 62.778R_\gamma + 14.787 \\ D &= -6.467R_\gamma^4 + 24.496R_\gamma^3 - 34.449R_\gamma^2 + 21.483R_\gamma - 5.018 \\ E &= 0.5874R_\gamma^4 - 2.2303R_\gamma^3 + 3.1381R_\gamma^2 - 1.9581R_\gamma + 0.4575 \end{aligned} \tag{8}$$

For steeper slopes (β > 40°), the influence of the lower layer becomes negligible. The coefficient μ depends only on the parameter $\cos\beta$:

$$\mu = \frac{FS}{M_{\phi,\beta}} = \begin{cases} a_2 + b_2\cos\beta + c_2(\cos\beta)^2 + d_2(\cos\beta)^3, & 40° < \beta \leq 60° \\ a_3 + b_3\cos\beta + c_3(\cos\beta)^2 + d_3(\cos\beta)^3, & \beta > 60° \end{cases} \tag{9}$$

with coefficients a_2 = 2.002152342, b_2 = -4.758057634, c_2 = 7.613352378, d_2 = -4.059692958, a_3 =13.39301464, b_3 = -85.44227782, c_3 = 225.1972591 and d_3 = -207.6079677.

The final surrogate model for the factor of safety (FS_{PM}) is obtained directly as $FS_{PM} = \mu M_{\phi,\beta}$, using the appropriate μ and coefficients.

6 MODEL VALIDATION

The predictive performance of the proposed surrogate model was evaluated by comparing the factor of safety predicted by the formulation (FS_{PM}) with the reference values obtained from limit equilibrium analyses conducted using the Morgenstern–Price method implemented in SLOPE/W (GeoStudio).

The validation dataset spans the entire parameter space considered in this study, including variations in slope inclination (β), friction angles of the upper and lower layers (ϕ_1 and ϕ_2), and unit weight ratios. This range of configurations encompasses both stable and near-failure conditions, as well as transitions between shallow and deep failure mechanisms, providing a comprehensive basis for evaluating the predictive capability of the surrogate formulation.

Figure 4 presents a direct comparison between the factor of safety predicted by the proposed method (FS_{PM}) and the reference values obtained from SLOPE/W (FS_{GS}). The results show an excellent correspondence between predicted and numerical values, with the data points closely distributed around the equality line. Linear regression analysis yields a coefficient of determination of approximately $R^2 \approx 1.0$, indicating an almost perfect correlation between the surrogate predictions and the reference solutions.

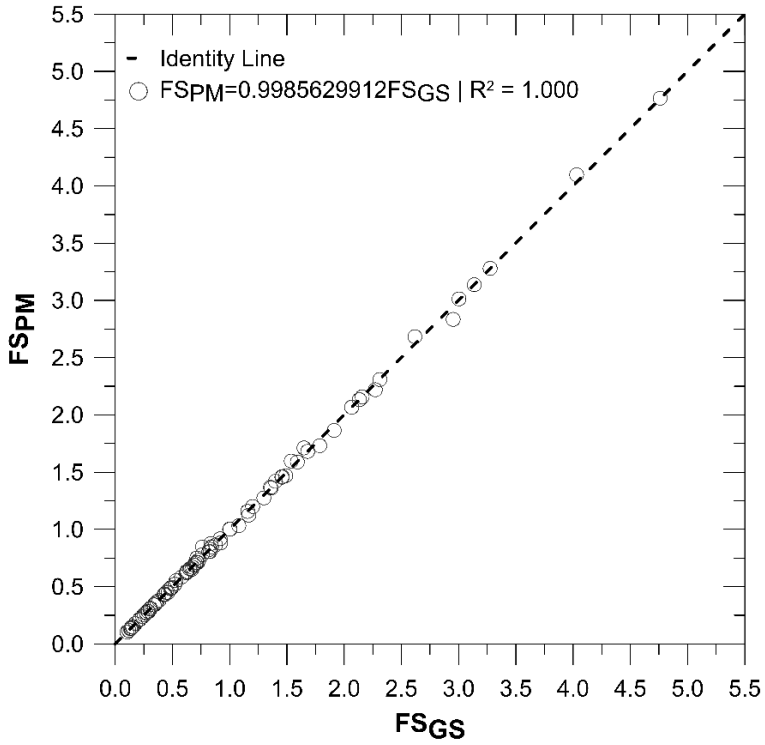


Figure 4. Comparison between the FS_{PM} and FS_{GS}.

Overall, the validation results confirm that the proposed surrogate formulation provides a reliable approximation of the Morgenstern-Price solutions while eliminating the need for iterative slip-surface searches. Consequently, the model offers a computationally efficient and physically consistent tool for rapid estimation of the factor of safety of two-layer frictional slopes.

7 FAILURE MECHANISM INTERPRETATION

The combined dimensionless parameter K not only governs the magnitude of the factor of safety but also provides an indicator of the expected failure mechanism (Figure 5). For slopes with $\beta \leq 40°$, two distinct failure mechanisms can be identified: 1) shallow failure ($K \leq 1$) the critical slip surface remains confined to the upper layer, and stability is mainly controlled by the upper-layer strength and slope geometry; 2) deep failure ($K > 1$) the slip surface penetrates into the lower layer, mobilizing the strength of both strata and making the stability response sensitive to interlayer strength contrast.

For steeper slopes ($\beta > 40°$), the failure surface remains confined to the upper layer regardless of strength contrast, indicating that slope geometry dominates the stability behaviour in this regime.

These results show that the parameter K effectively captures both the magnitude of the factor of safety and the transition between shallow and deep failure mechanisms in layered frictional slopes.

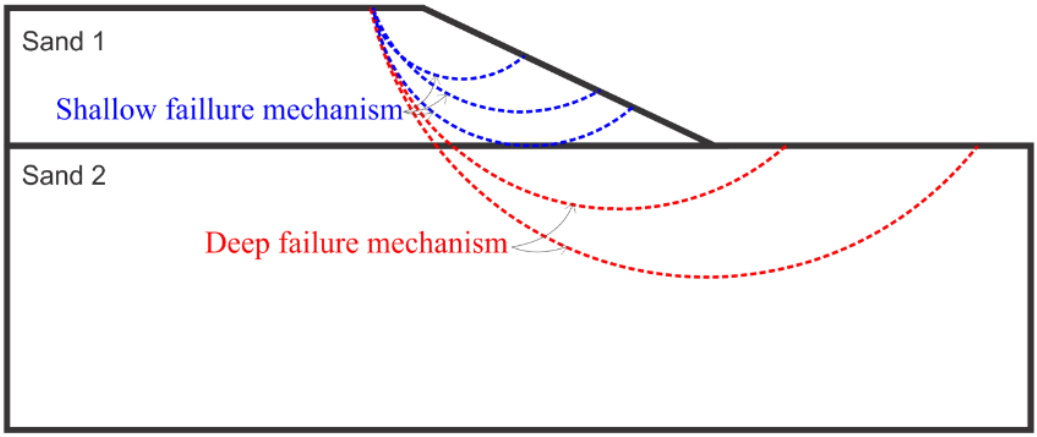


Figure 5. Representation of shallow and deep failure mechanisms in two-layer frictional slopes.

8 CONCLUSIONS

This study presented a surrogate analytical formulation for estimating the factor of safety of dry two-layer frictional slopes based on a comprehensive database of limit equilibrium analyses performed using the Morgenstern–Price method.

The stability behavior of layered frictional slopes was successfully described using three dimensionless parameters. For slopes with $\beta \leq 40°$, stability is influenced by the interaction between both soil layers, whereas for steeper slopes the failure surface remains confined to the upper layer and stability is primarily governed by slope geometry and upper-layer strength.

A combined dimensionless parameter was introduced to unify the stability behavior and capture the transition between shallow and deep failure mechanisms. Based on this parameter, expressions were derived to estimate the factor of safety directly, eliminating the need for iterative slip-surface searches.

Validation against numerical simulations demonstrated excellent agreement between the surrogate predictions and the reference solutions obtained from SLOPE/W. These results confirm that the proposed equations accurately reproduce the stability response of two-layer frictional slopes across a wide range of geometric and mechanical conditions.

Overall, the proposed formulation provides a simple, computationally efficient, and physically interpretable tool for rapid stability assessment of layered slopes. In addition to enabling direct estimation of the factor of safety, the proposed dimensionless framework also offers insight into the governing failure mechanisms associated with layered soil systems.

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